Real-Time Ramp Metering: High-Order Sliding Mode Control and Differential Flatness Concept

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Abstract
Real-time isolated freeway ramp metering is tackled in this paper, using High-Order Sliding Mode Control (HOSMC) and the concept of differentially flat systems. HOSMC is a robust control method more adaptive to uncertain systems such as the traffic one. On the other side, differential flatness is characterized by the fact that the whole system behaviour is described by the trajectory of a so-called “flat output” and a number of its successive time derivatives. This leads to a simple design of the open loop control, without integration of any differential equation. Numerical simulations using real-field data of a part of French Freeway sections around Paris have demonstrated the relevance of the proposed approach.

Keywords
High-Order Sliding Mode Control; Differential Flatness; Ramp-Metering; Freeway Control

Introduction
Intelligent transportation systems, including automated devices and advanced control techniques in surface transportation freeway infrastructures represent the most efficient solution to alleviate the daily impacts of traffic congestion [Kostialos, A. and Papageorgiou, M. 2004].

Traffic congestion appears when several vehicles attempt to use a common infrastructure with a limited capacity [Smaragdis, E., Papageorgiou, M. and Kosmatopoulos, E. 2004].

Traffic control also represents the main way to improve the freeway throughput and to ensure an efficient, safety and less pollutant transportation of goods and persons [Kostialos, A. and Papageorgiou, M. 2004]. Furthermore it contributes to a large reduction of direct and indirect costs.

Freeway traffic control can be achieved via a set of actions and measurements such as: dynamic speed limits, route guidance, ramp metering, etc.¹

Ramp metering also called admissible control, represents the most direct and efficient way to improve the freeway capacity by regulating the ramp flow at its entrance. As stated [Smaragdis, E., Papageorgiou, M. and Kosmatopoulos, E. 2004], ramp metering strategies are a valuable tool for efficient traffic management that can be classified into: Reactive strategies that aim to maintain the traffic conditions in freeway close to pre-specified set values using real-time measurements, and Proactive strategies aiming at specifying optimal traffic conditions for a whole freeway network based on demand and model prediction over a time horizon.

Ramp metering can be implemented locally (isolated ramp metering) in the vicinity of each ramp to calculate the corresponding ramp metering values. It can be implemented simultaneously (coordinated ramp metering) when the objective is to use available traffic measurements from larger freeway sections.

The first developed ramp metering algorithms are based on the classical linear control theory. The controllers are then designed using proportional P and/or proportional-integral PI regulators. Nevertheless, the nonlinear and uncertain character of the traffic flow calls for more robust and sophisticated algorithms. Among these algorithms, sliding mode control (SMC) represents a very powerful and robust method more appropriate for systems with uncertainty [Utkin, V. 1992], [Rouchon, P. 2005]. On the other hand, the differential flatness (proposed by Fliess and co-workers) [Fliess, M. Lévine, J. Martin, P. and Rouchon, 1 See, e.g. [Papageourgiou, M. Diakaki, C. and Dinopolou, A, 2003] for a detailed survey of the traffic control strategies.
P., 1995], [Rouchon, P., 1995] is a structural property that many dynamical systems, like freeway traffic flow one, exhibit. Differentially, flat systems enjoy the property of possessing a finite set of differentially independent outputs (i.e., outputs which do not satisfy, by themselves, nonlinear equations), called flat outputs, or linearizing outputs, such that all variables in the system (state and control input variables) are expressed in terms of these flat outputs and a finite number of their time derivatives [Fliess, M. Lévine, J. Martin, P. and Rouchon, P., 1995]. Furthermore, the flat outputs can be written in terms of differential functions of the system state variables and, possibly, a finite number of the control input derivatives. As stated in [Sira-Ramirez, H. and Agrawal S.K., 2004], differentially flat systems are dynamical systems which are linearizable to controllable linear systems by means of endogenous feedback, i.e. one that does not require external variables to the system to be completely defined. This leads to a simple controller design without integration on any differential equation.

Remark 1: As mentioned in [Sira-Ramirez, H. and Agrawal S.K., 2004], it’s important to under-line that one common misconception is that flatness amounts to dynamic feedback linearization. It is true that any flat system can be feedback linearized using dynamic feedback (up to some regularity conditions that are generically satisfied). However, flatness is a property of a system and does not imply that one intends to then transform the system, via a dynamic feedback and appropriate changes of coordinates, to a single linear system [Martin Ph., Murray R.M., and Rouchon, P. 1997].

In previous work [Iordanova, V. Abouaïssa, H and Jolly, D, 2008] the ramp-metering problem was tackled successfully using the first order sliding mode control and the concept of differential flatness. The purpose of this paper is to show that the combination of High Order Sliding Mode Control (HOSMC) [Levant, A., and Friedman, 2002] method with differential flatness principle can be applied for straightforward design of freeway ramp metering. Such approach seems to be a valuable control scheme which permits to achieve robust asymptotic output tracking especially for uncertain systems, e.g. uncertainty due to modelling errors and dynamic disturbances.

The paper is organized as follow: after recalling the principle and method of HOSMC and differentially flat systems, we present the freeway ramp metering algorithm then numerical simulations have been provided along with the revealed relevance of the proposed approach. Finally the main obtained results and some promising perspectives and further researches have been indicated.

### High Order Sliding Mode Control and Differentially Flat Systems

Sliding mode control is known to be a robust control method appropriate for uncertain systems such that the traffic one. Theoretical results as well as practical design examples show that high robustness is maintained against various kinds of uncertainties such as exogenous signal and measurement errors [Mammar Salim. Mammar Said and Netto M. 2006]. This control scheme is based on the concept of changing of the structure of the controller in response to the changing state of the system in order to obtain a desired value. A high speed switching control action is used to switch among different structures of the controller and the trajectory of the system is forced to move along a chosen switching manifold in the state space. The behaviour of the closed loop system is thus determined by the sliding surface [Utkin, V. 1992].

SMC is characterized by the following advantages:

- SMC is insensitive to system’s parameters variation and to external perturbations as well as modelling errors,
- The dynamic behaviour of the system may be tailored by the particular choice of switching function.

Since the first order SMC that may be implemented only if the relative degree of the sliding surface \( S \) is equal to 1, several SMC extensions have been developed (see e.g. [Manish, V. 2004]). In this paper, the focus is on the high order SMC and the concept of differential flatness. The principle of the proposed approach (called also a two degree of freedom design) is illustrated in Fig. 1.

![FIG. 1 A SYSTEMATIC DESIGN APPROACH PRINCIPLE](image)

The trajectory planning which yields a nominal control design is achieved using the flatness approach. The HOSM controller ensures the closed loop. The following section recalls some principle of these two concepts.
**High Order Sliding Mode Control**

High or arbitrary-order sliding (r-sliding) controllers with finite-time convergence have been only recently demonstrated. The control influence is a discontinuous function of the output and its $r-1$ real-time calculated successive derivatives. Increasing the system’s relative degree artificially, sliding control of arbitrary smoothness order can be achieved, completely removing the chattering effect.

Let us consider a smooth dynamic system $\dot{x} = f(x)$ with a smooth output function $y$. Assume that the system is closed by some possibly dynamical discontinuous feedback. Then, provided that successive total time derivatives $s, \dot{s}, \ddot{s}, \ldots, s^{(r-1)}$ are continuous functions of the closed-system state space variables, and the r-sliding point set (See e.g. Levant, A. and Fridman, L. 2002):

$$s = \dot{s} = \ddot{s} = \ldots = s^{(r-1)} = 0$$

is non-empty and consists locally of Filippov trajectories, the motion on above set is called r-sliding mode (r-order sliding mode). The idea of r-sliding controller is that the primary sliding mode disappears at the moment when the secondary one is to appear.

**Differentially Flat Systems**

The concept of Flat systems was introduced by Fliess and co-workers [Fliess, M. Lévine, J. Martin, P. and Rouchon, P., 1995]. [Fliess, M. Lévine, J. Martin, P. and Rouchon, P., 1992] more than 10 years ago. Differentially flat systems constitute a class of dynamical systems which represent the simplest possible extension of controllable linear systems to the nonlinear systems domain [Sira-Ramirez, H. and Agrawal S. K., 2004].

The flatness-based concept was developed in a differentially algebraic context and later expressed using Lie-Bäcklund transformation [Fliess, M. Lévine, J. Martin, P. and Rouchon, P., 1999].

Flat systems have a (fictitious) flat output, which can be used to explicitly express all states and inputs in terms of the flat output and a finite number of its derivatives. More precisely, the entire system behaviour is determined by the trajectory of a finite collection of quantities: flat outputs. This leads to a simple and elegant trajectories design. For a given system, the number of flat outputs is equal to that of the system inputs. The flatness concept is closely related to the state feedback linearization.

Notice that the flatness-based control methods may be expected to play a very significant role in high technology applications in the next few years, similar to what happened for nonlinear control in the last decade [Sira-Ramirez, H. and Agrawal S. K., 2004], [Rudolph, J. 2003].

**Flat Systems Definition**

In this section, we just sketch a tutorial definition of flatness for state-space control system. Consider the smooth system defined using the following equation:

$$\dot{x} = f(x, u) ; \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m$$

where $x = (x_1, x_2, \ldots, x_m)$ vector of state variables and $u = (u_1, u_2, \ldots, u_m)$ m scalar control. The system (2) is flat if and only if, $m$ real smooth functions $h=(h_1, h_2, \ldots, h_m)$ exist depending on $x$ and a finite number of $u$ derivatives, says $\beta$, such that, generically, the solution $(x,u)$ of the square differential-algebraic system ($t \rightarrow y(t)$ is given)

$$\dot{x} = f(x, u) ; \quad y(t) = h(x,u,\dot{u},\ldots,u^{(\beta)})$$

does not involve any differential equation and thus being of the form:

$$x = \Phi(y,\dot{y},\ldots,y^{(\beta)}), \quad y = \Psi(y,\dot{y},\ldots,y^{(\beta+1)})$$

where, $\Phi$ and $\Psi$ are smooth functions, and $\beta$ is some finite number [Rouchon, P. 1995]. The quantity $y$ is of fundamental importance: called “flat output” or linearizing output. In the control language, the flat output $y$ is such that the inverse of (3) has no dynamics [Isidori, A. 1986]. Differentially flat systems are very useful in situations where the explicit generation of trajectories is required. Since the behaviour of the flat systems is determined by the flat outputs, one can plan the trajectories in the outputs space and then connect these to appropriate inputs. More precisely, from the trajectories of the flat outputs $y$, we can deduce immediately the trajectories of the state $x$ and the input $u$ variables. Applications of the flatness concept to problems of engineering field have grown steadily in recent years and a variety of case studies has been shown to be flat and flatness based controllers based on trajectories generation by polynomial interpolation and then closing the loop on the obtained trajectories has been developed.

In the following section, we demonstrate how the jointly use of HOSMC and differential flatness can be useful in the traffic flow area and more precisely for real-time isolated ramp metering.
Traffic Flow Ramp Metering

Let us illustrate the principle of the proposed approach considering the following freeway section depicted in (Fig. 2), where, $\rho$ represents the traffic density in vehicles per kilometre per lane (veh/km/lane), $q_e$ and $q_s$ represent the input and the output flows veh/h (vehicles per hour), respectively.

\[ d_0 \text{ (veh/h)} \] is the traffic demand at the on-ramp origin and $\omega$ is the queue length in vehicles. The following equations describe the traffic evolution.

The conservation law (5) reads:

\[ \rho(t) = \frac{1}{L} \left[ q_e(t) - q_s(t) + q_r(t) \right] \tag{5} \]

where $L$ is the segment length. $q_r(t)$ is the metered on-ramp flow.

The output flow (6) is equal to the product of the traffic density, the mean speed $v$ and the number of lanes section $\lambda$.

\[ q_s(t) = \rho(t)v(t)\lambda \tag{6} \]

The above equations are supplied by a third expression describing the relationship between the three aggregated variables and defining the so-called fundamental diagram Fig. 3. Such diagram depicts the fluid zone and the congested one.

Ramp Metering Principle

The ramp metering principle consists of the regulation of the on-ramp flow (using traffic lights) in order to maintain the traffic density at the mainstream section around a critical value $\rho_c$ (veh/km/lane) (See Fig. 3, which corresponds to the maximum capacity $q_{\text{max}}$).

The output from the on-ramp origin depends on the traffic flow conditions on the mainstream section, for a metered on-ramp, of flow rate $r(t)$, $r(t) \in [r_{\text{min}}, r_{\text{max}}]$ (the control variable)². $r_{\text{min}}$ represents the minimum admissible ramp flow. $r_{\text{max}}$ is the ramp flow capacity (See e.g. [Papageorgiou, M. Blosseville, J.M and Haj-Salem, H. 1990], [Papageorgiou, M. Haj-Salem, H. and Middleham, F. 1997.]).

In order to avoid the wind-up effect, the calculated ramp flow $r(t)$ is truncated if it exceeds a range $[r_{\text{min}}, r_{\text{max}}]$ [Smaragdis, E. Papageourgiou, M. 2003].

Ramp Metering Algorithm

As already noticed in [Abouaïssa, H, Iordanova, V. and Jolly, D., 2006] and [Dryankova, V, H. Abouaïssa and Jolly, D. 2011], the considered motorway stretch (Fig. 2) is flat with $y = \rho$ its flat output. It is followed that all system variables can be expressed in terms of the flat output and its first time derivative:

\[ \left\{ \begin{array}{l}
y(t) = \rho(t) \\
r(t) = L\dot{y}(t) + y(t)v(t) - q_e(t); q_s(t) = y(t)v(t) \end{array} \right\} \tag{7} \]

According to differential flatness definition, the control variable in (7) is obtained thanks to system’s inversion.

The equation of the state variable allows choosing a suitable trajectory of the density (the flat output). The expression of the control (input) variable allows adding additional constraints to this density trajectory. This means that all-important properties of the system (5) are contained in such a differential parameterization.

Trajectory planning: the equation (7) corresponds to the open loop control algorithm. In order to define the trajectory planning, a suitable desired trajectory $y^*$ has to be defined. According to the expression of the control variable in (7), this trajectory must have smooth derivatives up to order two. From the initial and final conditions of the density \( \left( y(t_i) = y_i, \dot{y}(t_i) = 0 \right) \) and \( \left( y(t_f) = y_f, \dot{y}(t_f) = 0 \right) \), one can build this reference trajectory for the density (flat output) using a polynomial interpolation because of the reduced computational effort in the real time.

² As in [Smaragdis, E., Papageorgiou, M. and Kosmatopoulos, E. 2004], if the on-ramp is unmetered, $r(t) = 1$. 

environment [Sira-Ramirez, H. and Agrawal S.K., 2004]. Indeed, since at the system equilibrium points the constant values of the output variable \( \rho(t) \) and the flat output, \( y(t) \), perfectly coincide, one pose ourselves, instead of the original problem, the equivalent problem of controlling or transferring along a desired trajectory \( y^*(t) \), the flat output between the given initial and final equilibrium. One desires, then, to transfer the flat output between the values \( y^*(t_i) = y_i \) and \( y^*(t_f) = y_f \).

Using the polynomial interpolation, this is accomplished by prescribing the following desired trajectory for the flat output \( y \):

\[
y^*(t) = \begin{cases}
y_i & \text{for } t < t_i \\
y_i + (y_f - y_i)\sigma(t,t_i,t_f) & \text{for } t_i \leq t \leq t_f \\
y_f & \text{for } t > t_f
\end{cases}
\]

where \( \sigma(t,t_i,t_f) \) is a polynomial function of time, exhibiting a sufficient number of zero derivatives at times, \( t_i \) and \( t_f \), while as well satisfying: \( \sigma(t_i,t_i,t_f) = 0 \) and \( \sigma(t_f,t_i,t_f) = 1 \). Because for the studied system, we have four conditions, then a degree 3 polynomial is required as in Fig. 4:

\[
\sigma(t) = \begin{cases}
0 & \text{if } t \leq t_i \\
3\left(\frac{t-t_i}{t_f-t_i}\right)^2 - 2\left(\frac{t-t_i}{t_f-t_i}\right) & \text{if } t_i \leq t \leq t_f \\
1 & \text{if } t > t_f
\end{cases}
\]

In this way with the expression of the control variable in (7) and the relation (8), the nominal open loop control can be calculated:

\[
r^*(t) = Ly^* + q_e(t) - q_e(t)
\]

High order sliding mode trajectory tracking: Endogenous feedback control strategies specified on the basis of desirable linearly decoupled behaviour of the linearizing output are generally non-robust with respect to un-modelled external perturbations and bounded parametric variations [Manish, V. 2004], [Isidori, A. 1986]. Then HOSMC is applied in order to provide a suitable and implementable alternative for decoupled linearization of the flat system.

Starting from the freeway stretch depicted in (Fig. 2), it is recalled that the main objective of the control is to maintain the mainstream density below a critical value. As stated above, the considered system is flat with \( y = \rho \) its flat output. Consider that the sliding surface is determined by this flat output.

\[
s = y - y^*
\]

\( y^* \) is the reference trajectory (the desired density). The first derivative of \( s \) reads:

\[
\dot{s} = \dot{y} = \frac{1}{L}(q_e - yv + r)
\]

Remark that the control \( r \) appears in the first derivative. One can therefore consider to develop a controller of order 2 for example. In this case, the switching control \( r \) is moved to the first derivative \( \dot{r} \). This leads to complete chattering elimination. The control law is designed using the "Algorithm with a prescribed convergence law" (see e.g. [Levant A. and Fridman, L. 2002] and [Utkin, V. 1992] for more details about such algorithm). In this case, one can obtain the auxiliary control:

\[
\xi = -\lambda_s \text{sign} \left( \dot{s} + \gamma \sqrt{2} \text{sign}(s) \right)
\]

This allows obtaining the ramp control equation as follow:

\[
r = L \left[ -\lambda_s \text{sign} \left( \dot{s} + \gamma \sqrt{2} \text{sign}(s) \right) \right] + yv - q_e
\]

The equation above (11) is the ‘state feedback’ high order sliding mode control based on differential flatness for isolated ramp metering.

**Numerical Simulation**

**Site Description**

In order to evaluate the performance of the proposed approach, a stretch of the French freeway A6W is taken into consideration depicted in Fig. 5.

The A6W freeway network is located in the southern part of the Ile de France and managed by the DiRIF.

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3 For the polynomial calculation see e.g. [Rudolph, J. 2003], [Sira-Ramirez, H. and Agrawal S.K., 2004].

4 Acronym of Direction Interdépartementale des Routes
As stated in [Boukhnifer, M. and Haj-Salem, H. 2010], this site is the most critical part of the Île de France freeway network. The studied section length covers around 20 km including five on-ramps and three off-ramps.

**FIG. 5 A6W FREEWAY FIELD TEST SITE**

Remark 2: We restrict our study to the following three on-ramps: “Ris-Orangis”, “Grigny” and “Viry” (See e.g. Fig. 5) (See [Haj-Salem, H, Farhi, N. and Lebacque, J.P. 2012] for more details).

The length of the considered site, A6W, is about 8 km. The Automatic Data Collection is done using a set of stations (RAD). Each RAD is based on electromagnetic loops. All the freeway network of the Di RIF is equipped with electromagnetic loops installed every 500 meters. They allow the measurement of the flow \( q \) and the occupancy “TO”. Double loops are implanted every 3 or 4 km for speed measurement [Haj-Salem, H, Farhi, N. and Lebacque, J.P. 2012]. Measurements of speed and occupancy allow the estimation of the electrical length of vehicles and then, the calculation of the mean speed on the surrounding simple loops and possibly estimate the rate of the Trucks in the segment.

The supervision of RAD is provided by the new module, called “KIR”, which ensures the classification of the collected data and the restoration of the missing one.

The used data extracted from the database of the operating system SIRIUS (DIRIF) correspond to the day “Tuesday, March 16, 2010”. They consist of average values of flow, occupancy and speed, aggregated over a time interval of 6 minutes for each station.

For the on-ramps, two stations are installed. The first one is located at the vicinity of the ramp behind the signal light, and used as actuator to realize the on-ramp metering calculations. The second one, situated at the end of the on-ramp, allows activating the override tactic when the on-ramp is full.

**Summary Description of the Used Traffic Model**

For numerical simulations, the second order macroscopic model was used [Payne, H.J. 1971], [Papageorgiou, M. Blosseville, J. M and Haj-Salem, H. 1990]. For a given freeway section \( i \) the expression (5) becomes:

\[
\dot{\rho}_i = \frac{1}{L_i} \left[ q_{i-1} - q_i + r - b \right] 
\]

where \( b \) is the off-ramp flow. Equation (12) is completed by the relationship (13) between the traffic volume \( q_i \) and the mean speed \( v_i \).

\[
q_i = \rho_i v_i
\]

and the momentum (or speed) equation (14).

\[
\dot{v}_i = \frac{1}{\tau_i} \left[ V_e (\rho_i) - v_i (t) \right] + \frac{1}{\tau_i} v_i (t) \left[ v_{i-1} (t) - v_i (t) \right]
\]

Equation (14) regards the mean speed as an independent variable in order to take into account the traffic flow dynamics, in terms of relaxation, convection and anticipation (See e.g. [Papageorgiou, M. Blosseville, J. M and Haj-Salem, H. 1990] and the references therein for more details about the second order macroscopic models).

\[
V_e (\rho_i) = v_f \exp \left[ -\frac{1}{a} \left( \frac{\rho_i}{\rho_{cr}} \right)^a \right] 
\]

Additionally, in order to take into account the impact of the entering from the on ramp, traffic volumes, we may add the following expression to equation (14):

\[
\frac{\Delta}{L_i} \frac{\eta_i v_i}{\rho_i^+ + \kappa}
\]

\( r_i, v_i, \) are the control variable, and the mean speed, respectively. \( \tau, \nu, \kappa, \delta, \) are model parameters.

Finally, the calculated, from the algorithm, volumes
are submitted to a set of queue length constraints. Then, the flow $q_r(t)$ is:

$$q_r(t) = r(t) \dot{q}_r(t)$$

(17)

$$\dot{q}_r(t) = \min \left[ d_o + \frac{\omega}{T} Q_{sat} \min \left( 1 - \frac{\rho_{max} - \rho}{\rho_{max} - \rho_c} \right) \right]$$

(18)

$\rho_{max}$ is the maximum density. $Q_{sat}$ is the on-ramp capacity (veh/h).

**Simulation Results**

The first step before conducting numerical simulations is to solve the problem of calibrating the used model with the field real data. This yields the following main parameters values (See Table 1)7:

<table>
<thead>
<tr>
<th>On-ramps</th>
<th>Length (m)</th>
<th>$v_f$</th>
<th>$\rho_{cr}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RisOrangis</td>
<td>600</td>
<td>101.164</td>
<td>26.827</td>
<td>2.11</td>
</tr>
<tr>
<td>O-Grigny</td>
<td>1410</td>
<td>119.931</td>
<td>21.696</td>
<td>2.851</td>
</tr>
<tr>
<td>O-Viry</td>
<td>750</td>
<td>117.708</td>
<td>42.49</td>
<td>1.1934</td>
</tr>
</tbody>
</table>

Figures 7 and 8 represent the traffic densities and mean speed time evolutions for the on-ramp “RisOrangis” with the traffic demands depicted in Figure 6.

Fig. 9a and Fig. 9b show the densities evolutions for the two cases for “Grigny” and “Viry” locations.

7 For simplicity’s sake, table 1 presents just the model parameters necessary for the fundamental diagram, other parameters such as $\tau$, $v$, $\kappa$, ..., are not presented.
systems (See Fig. 9-11) acts favourably in order to minimize the congestion impact.

FIG. 10 CONTROL VARIABLE TIME EVOLUTION

Quantitative Results

Performance evaluation from the quantitative viewpoint was conducted (See [Dryankova, V., Abouaïssa H., Haj-Salem, H. Jolly, D. 2011], [Dryankova, V., Abouaïssa H., Haj-Salem, H. Jolly, D. and Nikolov, E. 2012]) based on the computation of the criteria defined in (19), (20) and (21).

\[
TTS = T \sum_{k=1}^{N} \sum_{i=1}^{L} \rho_i(k) \quad (19)
\]

where,

- \(TTS\) the total time spend expressed in vehicles *h
- \(\rho_i(k)\) : traffic density at the location \(i\)
- \(L_i\) : segment length \(i\)
- \(N\) : total number of segments
- \(k\) : time horizon
- \(T\) : simulation step

\[
DTP = T \sum_{k=1}^{N} \sum_{i=1}^{L} q_i(k) \quad (20)
\]

\(DTP\) (Table 2) represents the total travelled distance in vehicles*km, where \(q_i(k)\) is the measured flow at the location \(i\).

\[
VM = \frac{DTP}{TTS} \quad (21)
\]

with \(VM\) the generalized mean speed expressed in km/h.

Other environment criteria have been computed such as the fuel consumption (liters:l). The consumption index expressed in vehicles*l, is computed over the time horizon using the speed measurements on the freeway axis.

\[
Consom = \sum_{k}^{T} \frac{T}{100} \sum_{m}^{100} \sum_{i}^{L}
\]

and the Emission of CO (Carbon Monoxide) and HC (Hydrocarbon) [Haj-Salem, H, Farhi, N. and Lebacque, J.P. 2012].

In terms of the criteria, Fig. 11 and Fig. 12, show the total benefits based on total travel spent and total travelled distance.

Table 2 summarizes the quantitative results which demonstrate the relative benefit of the proposed algorithm.

In terms of the environment criteria, with respect to fuel consumption, the gain compared with the no control case is about −9%. The gains in terms of HC and CO criteria are of −7% and −11%, respectively.

FIG. 11 TTS CRITERION EVOLUTION: – NO-CONTROL, – CONTROL CASE

FIG. 12 DTP CRITERION EVOLUTION: – NO-CONTROL, – CONTROL CASE
TABLE 2 QUANTITATIVE RESULTS

<table>
<thead>
<tr>
<th>O-N441-RisOrangis</th>
<th>No-Control Case</th>
<th>HOSMC</th>
<th>Gain%</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTS</td>
<td>6700</td>
<td>6265</td>
<td>-6.6</td>
</tr>
<tr>
<td>DTP</td>
<td>120460</td>
<td>120460</td>
<td>-</td>
</tr>
<tr>
<td>VM</td>
<td>65</td>
<td>70</td>
<td>6.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>O-Griny</th>
<th>No-Control Case</th>
<th>HOSMC</th>
<th>Gain%</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTS</td>
<td>14600</td>
<td>12300</td>
<td>-15.6</td>
</tr>
<tr>
<td>DTP</td>
<td>530250</td>
<td>530250</td>
<td>-</td>
</tr>
<tr>
<td>VM</td>
<td>65</td>
<td>70</td>
<td>6.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>O-Viry</th>
<th>No-Control Case</th>
<th>HOSMC</th>
<th>Gain%</th>
</tr>
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<tr>
<td>TTS</td>
<td>11577</td>
<td>10775</td>
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<tr>
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<td>-</td>
</tr>
<tr>
<td>VM</td>
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</table>

Conclusion

The paper deals with the combination of the HOSMC and differential flatness concept in order to solve the problem of isolated traffic ramp metering. The suggested control algorithms are based on the system inversion carried out using the flatness-based concept free from integration of any differential equations. This setting has been then used for trajectories generation of the traffic densities. Having these planned trajectories, the high order sliding mode control was used in order to track and stabilize on-line any deviation of the prescribed trajectories. The combination of differential flatness with the HOSMC technique is qualified as a valuable control scheme. Simulation results, carried out using real field data of the French freeway segment with 3 on-ramps, demonstrate the relevance of the proposed approach. The quantitative evaluations show that such algorithm decreases significantly the Total Time Spend and increases the Mean Speed compared with the no control case. The environmental indices show a relatively high gain in terms of fuel consumption, HC and CO emissions.

The authors will further exploit the principle of the proposed method and the possibility to extend it to more large and complex networks for coordinated ramp metering as well as integrated control combining ramp metering and other traffic flow control measurements such as dynamic speed limits.

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