Effects of Yield Strength and Elastic Modulus on Cement Sheath Interface Stress at Well Head

Jingfu Zhang*, Siyu Chen, Yingbo Lv, Jundong Chen, Qiang Zhang
Petroleum Engineering Institute, Northeast Petroleum University, Daqing 163318, China
*zjf286@126.com

Abstract
For reasonable choice of mechanical parameters and keeping long-term sealing performance of cement sheath, effect laws of yield strength and elastic modulus on the interface stress of cement sheath at well head were researched. Finite element mechanical model of the casing-cement sheath-stratum combination was established. Damage conditions of cement sheath structural integrity were analyzed. Results showed that every interface stress increased with the increase of the yield strength while cement sheath yielding deformation occurred. And the yield strength could not affect all interface stresses while the deformation was elastic. For lower elastic modulus, elastic deformation occurred in cement sheath and every interface stress increased with the increase of elastic modulus. For high elastic modulus, Cement sheath was partially or fully yielded. With the increase of elastic modulus, the interface contact stress increased, the inner interface circumferential stress decreased, and the interface contact stress was turned into tensile stress after unloading.

Keywords
Cement Sheath; Yield Strength; Elastic Modulus; Interface Stress; Combination

Introduction
After cementing, the sealing efficiency of cement sheath would be damaged by stress generated inner the cement sheath or at its interface from producing load. It was important for reasonable designing cement slurry system to analyze the change law of the cement sheath stress. Experts and scholars such as Rodriguez W J, Li Zifeng, Li Jun, Yin Youquan ect. [1-7] had researched the mechanical problems of the casing-cement sheath-stratum combination. Different combination mechanical models were established. Influence law of formation and cement mechanical parameters on the combination were analyzed. However, conditions of continuous radial deformation for the combination were assumed in above achievements of mechanical model and numerical calculation. The problem of probable damage of cement sheath body and bond interface by tensile stress were ignored. Based on the mechanical analysis in the literature [7], the influence law of the cement sheath yield strength, elastic modulus on its interface stress were deeply analyzed.

Finite Element Mechanical Model

FIGURE 1. FINITE ELEMENT MECHANICAL MODEL
Casing and stratum were bonded together to form the combination of casing-cement sheath-stratum by cement sheath, which was formed by hydration and hardening of cement slurry pumped into the oil and gas well annulus. According to the literature [7] for condition of the creep formation, PLANE183 plane strain unite could be used to describe the mechanical action among casing-cement sheath-stratum combination when casing bending instability and failure were not considered. The 1/2 model could be selected to analyze the stress and strain mechanical problems of the combination for loading and unloading. The mechanics function machine of finite element mechanical model was shown in Fig. 1. In the mechanical model, the linear elastic and nonlinear plastic deformation of cement sheath were all taken into account under the production load action. And the carrying capacity of cement sheath bond interface was also considered to establish the model.

**Constitutive Equation of Cement Sheath**

According to the experiment results, deformation properties of cement sheath were the same as those of elastic-plastic material under tri-axial stress action (Fig.2). Elastic deformation was generated at the initial loading stage. Plastic deformation occurred when the loading stress was higher than cement sheath yield strength. According to the deformation characteristic given in Fig.2, cement sheath could be assumed as ideal elastic-plastic material and the following formula could approximately be used to describe the mechanical constitutive relationship for three-dimensional stress condition:

\[
\begin{align*}
\sigma &= E\varepsilon & 0 \leq \varepsilon \leq \varepsilon_e \\
\sigma &= \sigma_e & \varepsilon_e \leq \varepsilon \leq \varepsilon_s
\end{align*}
\]

Where, \( \sigma \) was stress; \( \varepsilon \) was strain; \( \sigma_e \) was cement sheath yield strength; \( \varepsilon_e \) was the strain at yield point; \( E \) was cement sheath elastic modulus; \( \varepsilon_s \) was total strain generated at cement sheath damage.

**Conditions of Combination Failure**

Structure integrity damage of casing-cement sheath-stratum combination depended mainly on the cement sheath body and cement bond interface, both of which were the relatively weak parts of combination's carrying capacity. If any one of them was damaged, the combination body was also damaged and the sealing efficiency of cement sheath was failure. Therefore, failure conditions could be established based on the carrying limits of cement sheath body and its bond interface.

There were two characteristics in the mechanical properties of the cement sheath by tri-axial stress action according to experimental results. One was that the plastic deformation would be produced when cement sheath was yielded. And another was that compressive strength was high but tensile strength was low. Thus, the distortion energy density theory and the maximum tensile stress theory could be used to determine the cement sheath carrying capacity.

Under the conditions of the tri-axial stress, cement sheath yielded with Von Mises yield condition:

\[
\sigma_M \leq \sigma_e
\]

\[
\sigma_M = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}
\]
Where, \( \sigma_M \) was Von Mises stress; \( \sigma_1, \sigma_2, \sigma_3 \) were the three principal stresses. Cement sheath would suffer tensile failure when the tensile stress was higher than the maximum tensile strength:

\[
\sigma_{\text{max}} \leq \sigma_t
\]

Where, \( \sigma_{\text{max}} \) was the maximum tensile stress; \( \sigma_t \) was the tensile strength of cement sheath.

The cement sheath bond interfaces were the weak parts of the combination and were easily damaged by load action, because of that the bond strength were relatively low. When the contact stress generated at the interface was compressive stress, casing and cement sheath or cement sheath and stratum was at the squeezing state, which represented that the combination was contacted closely. But when the contact stress was tensile stress, the bond interface would be threatened by the radial stretch. This radial tensile stress was generally produced in the process of unload because of that cement sheath generated plastic deformation during loading and irrecoverable deformation occurred after unloading. Based on the cement sheath bond interface mechanical relation, bond interface began to tear when radial tension stress reached the cementation strength, and the contact stress was zero when bond interface was completely torn. So, two mechanical conditions decided whether the bond interface would contact or separate: interface contact stress was zero, and there was a gap in bond interface.

According to the above analysis, the combination structure had three aspects of damage conditions, which could be used to evaluate the structure integrity.

**Influences of Yield Strength**

Cement sheath yield strength was used to evaluate whether there was yielding deformation by load. Figure 3 showed the influence law of yield strength on interface stress calculated by mechanical model. Calculation parameters were: Borehole diameter was 241.3mm. Casing diameter was 177.8mm. Casing thickness was 12.65mm, elastic modulus was 210000MPa and poisson ratio was 0.3. Cement sheath elastic modulus was 10000MPa, poisson ratio was 0.178 and tensile strength was 3.5MPa. The first interfacial bond strength of the cement sheath was 0.6MPa and the second interfacial bond strength was 0.18MPa. Radius of the wellbore surrounding rock was 1m. The crustal stress was zero for the depth of stratum being 0 at well head. Rock elastic modulus was 24000MPa and poisson ratio was 0.25. Loading added at well head was 35MPa, which was the inner pressure of casing. Fig.3(a) was the calculated results after loading, curves 1 and 2 displayed interior and external interface contact stress of cement sheath (positive value was compressive stress); curves 3 and 4 were interior and external interface circumferential stress (positive value was tensile stress); curves 5 and 6 graphed interior and external interface Von Mises stress; Fig.3 (b) showed the calculated results after unloading, curves 7 and 8 were interior and external interface contact stress (negative value was tensile stress).

With the given load, the cement sheath yield strength and low crustal stress at well head and other conditions, when the casing was loaded, contact stress. Mises stress and circumferential stress of the cement sheath interior interface were higher than corresponding stress of the external interface. When loading (Fig.3 (a)), according to various forms of interface stresses changing with yield strength, it could be divided into two intervals. One was the zone of low yield strength (≤10MPa). Within this zone, the inner side of cement sheath yielded and got partially plastic deformation easily during loading, which Mises stress at yielding part equaled to yield strength (curve 5) based on equation (1). If load was high, whole cement sheath would yield. Each interface stress increased with increasing yield strength. Among all interface stresses, internal interface Mises stress (curve 5) and circumferential stress (curve 3) had obvious variation with yield strength, and others had relatively small change. Another was the zone of high yield strength (>10MPa), cement sheath got elastic deformation by loading, while, yield strength was higher than Mises stress. The yield stress could not affect cement sheath interface stress.

Curves 3 and 4 in Fig.3 (a) showed that when the yield strength reached or over 10MPa, circumferential tensile stress (3.82MPa) of cement sheath internal interface was higher than the given tensile stress (3.5MPa), tensile cracks would appear at the inside cement sheath. However, when the yield strength was lower than 10MPa, the stress was lower than the tensile strength and tensile cracks could not exist.

As shown in Fig.3 (b), as cement sheath only had elastic deformation during loading when the yield strength
exceeded 10MPa, the deformation would fully recover after unloading, at this time, interface contact stress, circumferential stress etc. all restore to the initial state of zero. When cement sheath yielded during loading (yield strength≤10MPa), because the complete recovery of cement sheath deformation was impossible, the radial interface contact stress would change from pressure stress of loading into tensile stress which probably tears cement sheath bond interface, because the boundary stratum could not provide sufficient impetus of deformation recovery. The higher the degree of yielding deformation was, the more easily tearing bond interface was produced. Under the calculated condition in this paper, when yield strength was lower than 10MPa, both of two bond interfaces stresses were tensile stress. The lower the yield strength was, the greater tensile stress was. For the yield strength was 6MPa, the inner interface of cement sheath was torn, while tensile stress was 0 and micro gap was 0.0013mm.

The results in Fig.4 showed that differences of the interior and external interface variation trends could also be classified according to elastic and plastic deformation: ① at the lower value stage of elastic modulus, cement sheath had elastic deformation. With elastic modulus increasing, both of two interfaces various stresses increased; ② when elastic modulus increased to a certain value, the interior interface Mises stress was no less than the yield strength; partial plastic deformation was generated inside the cement sheath, interface Mises stress equaled to the yield strength. After that, the interior interface Mises stress was no longer changeable (see curve 5 in Fig.4).
Effects of Yield Strength and Elastic Modulus on Cement Sheath Interface Stress at Well Head

Increasing elastic modulus, cement sheath interface contact stress, external interface circumferential tensile stress and Mises stress increased (curves 1, 2, 4 and 6 in Fig. 4), and the interior interface circumferential stress (curve 3) decreased.

The results showed that when the elastic modulus of the cement was 9000 MPa, interior interface Mises stress (9.97 MPa) was close to the yield strength, and when elastic modulus was 10000 MPa, cement sheath yielded at inner side, and the Mises stress would remain at a constant value (10 MPa). Meanwhile, due to partial yielding of cement sheath, not only did the value variation trend of circumferential stress change with elastic modulus (curve 3 in Fig. 4), but also the position of the maximum circumferential stress moved to inside cement (Fig. 5).

In Fig. 5, the elastic modulus of the cement was 12000 MPa. The circumferential stress value at different radial position was described by different color, which represented different stress value. The interior interface circumferential stress was 3.44 MPa, and the maximum internal tensile stress represented by red portion was 4.18 MPa.

According to Fig. 4 and Fig. 5, when the elastic modulus was 9000 MPa, the high tensile stress (3.96 MPa) was produced at cement sheath interior interface (curve 3 in Fig. 4) and the stress exceeded the tensile strength of cement sheath (3.5 MPa). So, interior interface tensile cracks would be generated. The structural integrity of the cement sheath would be damaged. Then, with elastic modulus continually increasing, even though the interior interface circumferential stress was decreased, it was still possible that the maximum circumferential stress (4.18 MPa) inside cement sheath was higher than the tensile strength, which led to tensile break of inside cement.

After unloading, for the elastic deformation region of low elastic modulus, casing, cement sheath and stratum generated elastic deformation recovery, all stresses returned to the initial condition before loading. But for the region of high elastic modulus, when cement sheath yielded, the interior and external interface contact stress was turned into tensile stress after unloading (Fig. 4 (b)), and compressive stress was generated at the circumferential direction of cement sheath. With increasing elastic modulus, interface radial tensile stress increased, and tearing cement sheath bond interface became more possible.

According to the above calculated results, cement sheath various mechanical parameters cooperated harmoniously when taking yield strength, elastic modulus and load capacity all into account. Therefore, working loading and cement sheath mechanical characteristics should be the basement to design cement slurry system.

Conclusions

(1) For the zone of low yield strength, when loading, internal cement sheath yielded and got partially plastic deformation easily. Within this zone, with increasing yield strength of cement sheath, each interface stress increased. Among all interface stresses, cement sheath internal interface Mises stress and circumferential stress have obvious variation with yield strength, and others have relatively small change. For the zone of high yield strength, cement sheath got elastic deformation. Yield strength could not affect cement sheath interface stress.

(2) At the initial stage of elastic modulus increasing, cement sheath had elastic deformation. With elastic modulus
increasing, various stresses of cement sheath interior and external interface increased. When elastic modulus increased to a certain value, partial plastic deformation was generated inside the cement sheath. After that, cement sheath interior interface Mises stress was no longer changeable. With increasing elastic modulus, cement sheath interface contact stress, circumferential tensile stress and Mises stress of external interface increased, and circumferential stress of interior interface decreased.

(3) In the process of cement sheath elastic deformation during loading, maximum circumferential tensile stress was at cement sheath interior interface. After unloading, the combination structure could fully recover, and all stresses could return to the initial condition. In the process of cement sheath yielding deformation during loading, position of maximum circumferential tensile stress moved from interior interface to inside cement sheath. After unloading, there was radial tensile stress on cement sheath interface, and bonding interface had potential risk of tearing.

ACKNOWLEDGMENT

The authors wish to thank National Natural Science Foundation of China and National Natural Science Foundation of Heilongjiang Province for supporting the research. The supporting research projects were respectively: Mechanical analysis and research on annular sealing reliability of cement sheath (51474074); Analysis on structure integrity of casing-cement sheath-stratum cementing combination (E201334).

REFERENCES