Optimal Contribution Rate of Public Pension in China within an OLG Model

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Abstract
Employing an overlapping-generations model, this paper investigates the urban public pension system in China to examine the effects of the pension contribution rates and population growth rate on the capital-labor ratio, social pool benefits, individual account principal, consumption and utility, and find the optimal firm contribution rate. The results are as follows: Raising the individual contribution rate increases the individual account principal; and raising the firm contribution rate increases the social pool benefits, whereas decreasing the capital-labor ratio, individual account principal, working-period consumption, retirement-period consumption and utility. The fall in the population growth rate increases the capital-labor ratio, individual account principal, working-period consumption and utility, whereas decreasing the social pool benefits and retirement-period consumption. The optimal firm contribution rate falls with the population growth rate. It will do more good than harm to raise the individual contribution rate, reduce the firm contribution rate and strictly implement the population policy.

Keywords
Public Pension; Contribution Rate; OLG Model

Introduction
The Chinese State Council Document 38 in 2005, “Decision on Improving Basic Pension System for Enterprise Employees”, was issued in December, 2005, in which a new public pension system was introduced in the urban areas: The government establishes an individual account for each employee and a social pool for all employees and retirees. Each firm contributes 20% of its payroll to the social pool, while each employee contributes 8% of her/his wage to her/his individual account. The social pool fund is used to pay the current retirees as pay-as-you-go (PAYG) pension benefits, while the accumulation in the individual account is used to pay the individual herself/himself when she/he retires as funded pension benefits. Each retiree receives funded pension benefits from her/his individual account and PAYG pension benefits from the social pool. Such a public pension system is a partially funded one.

The main goal of the document is to make the individual accounts have full real assets and pull off the social pool balance between revenues and payments. In the last decade, the social pool overdraw the individual accounts because the former was short of paying PAYG pension benefits. Consequently, the individual accounts had not full real asset accumulation as designed. Obviously, the new public pension system has effect not only on the social pool and individual accounts, but also on the capital-labor ratio, consumption and utility. In addition, the population growth rate has been falling because of China’s population policy. The social pool belongs to intergenerational transfers with PAYG type. Thus, the fall in the population growth rate also has effect on the pension contribution rate for the social pool.

The above effects can be examined by overlapping generations (OLG) model. Some of the literature on public pension with OLG model study PAYG pension system (e.g., Pecchenino and Pollard, 2002; Groezen et al., 2003). Some analyze fully funded pension system (e.g., Abel, 1987). Some investigate both PAYG and fully funded pension systems (e.g., Altig and Davis, 1993; Zhang et al., 2001). Samuelson (1975) studied the optimum social security in a life-cycle growth model; and adjusted the capital-labor ratio to the modified golden rule level to maximize the social welfare by controlling the social security taxes. The approach to find the optimal social security taxes is to equate the rate of interest to the growth rate of economy in a decentralized economy. Blanchard and Fischer (1989) elaborated the principle of social optimum. A social planner maximizes the social welfare by rationally allocating the social resources. This approach can be used to derive the optimal pension contribution rate in China’s partially funded public pension system.

This paper employs Blanchard and Fischer’s (1989)
OLG model, but the utility and production functions are specialized, to investigate China’s partially funded public pension; examines the effects of the individual contribution rate, firm contribution rate and population growth rate on the capital-labor ratio, social pool benefits, individual account principal, consumption and utility, and seeks the optimal firm contribution rate. It is interesting to show that raising the firm contribution rate increases the social pool benefits, whereas decreasing the consumption of the retirees; and the optimal firm contribution rate decreases with the population growth rate.

In the above literature, pensions are financed only by wage taxes, which are usually lump-sum taxes. However, in most of the countries that have public pension systems, the governments levy pension taxes on each employee’s wage and each enterprise’s payroll with proportional taxes. Thus in this model, as real life, the government levies pension taxes on each worker’s wage and firm’s payroll with proportional taxation.

The rest of this paper is organized as follows: Section 2 presents the model in market economy. Section 3 gives the comparative statics. Section 4 simulates the effects. Section 5 derives the optimal firm contribution rate and simulates the effect of population growth rate on it. Section 6 concludes the paper.

The Model

Based on Diamond (1965), this model extends that of Blanchard and Fischer (1989) by considering a partially funded public pension system combining social pool and individual accounts. There has a closed economy composed of numerous individuals and firms and a government. The generation born at the beginning of period t is called generation t. The population grows at the rate of $N_t = N_t - N_{t-1}$, where $N_t$ is the population size of generation t.

Individuals

Individuals live for two periods: working period and retirement period. In the working period, each individual earns wage by supplying inelastically one unit of labor, makes pension contributions, consumes part of her/his income, and saves the rest. In the retirement period, she/he consumes the savings with accrued interest, funded pension benefits and PAYG pension benefits.

Each individual derives utility from the working-period consumption, $c_{1t}$, and the retirement-period consumption, $c_{2t+1}$. The utility is described by an additively separable logarithmic function. Each individual maximizes her/his own utility:

$$\max \{c_{1t}, c_{2t+1}\} U_t = \ln c_{1t} + \theta \ln c_{2t+1}, \quad (1)$$

s.t.

$$c_{1t} = (1 - \tau)w_t - s_t, \quad (2)$$

$$c_{2t+1} = (1 + r_{t+1})s_t + (1 + r_{t+1})I_t + P_{t+1}, \quad (3)$$

where $\theta \in (0,1)$ is the individual discount factor, $w_t$ the wage, $\tau$ the individual contribution rate, $s_t$ the savings, $r_{t+1}$ the interest rate, $I_t$ the individual account principal per worker, and $P_{t+1}$ PAYG pension benefits. The first-order condition is

$$-c_{2t+1} + \theta(1 + r_{t+1})c_{1t} = 0. \quad (4)$$

This familiar expression implies that the utility loss from reducing one unit of working-period consumption is equal to the utility gain from increasing $(1 + r_{t+1})$ units of retirement-period consumption.

Firms

Firms produce a homogenous commodity in competitive markets. The production is described by Cobb-Douglas function $Y_t = AK_t^{\alpha}N_t^{1-\alpha}$ or $y_t = Ak_t^{\alpha}$, where $Y_t$ is the output in period t, $K_t$ the capital stock, $\alpha \in (0,1)$ the capital share of income, $A$ the productivity, $k_t = K_t/N_t$ the capital-labor ratio, and $y_t$ the output-labor ratio.

Firms make pension contributions at the rate of $\eta \in (0,1)$ on their payroll. Assume that capital depreciates entirely within one period in production. According to the product distribution, one can get that $AK_t^{\alpha}N_t^{1-\alpha} = (1 + r_t)K_t + (1 + \eta)w_tN_t$. Using Euler’s theorem gives:

$$1 + r_t = \alpha Ak_t^{\alpha-1}, \quad (5)$$

$$w_t = \frac{(1 - \alpha)Ak_t^{\alpha}}{1 + \eta}. \quad (6)$$

The Government

The social pool fund is paid to the retirees in the current period as PAYG pension benefits: $P_tN_{t-1} = \eta w_tN_t$ or
\[ P_t = (1 + n)\eta w_i. \]  
(7)

The accumulation in the individual account is used to pay the individual when he/she retires in the next period as funded pension benefits: 
\[ (1 + r_{\tau+1})w_i = (1 + r_{\tau+1})I_t \] or 
\[ I_t = \tau w_i. \]  
(8)

Dynamic Equilibrium

The savings and the individual account principal of the workers in period \( t \) generate the capital stock in period \( t+1 \):
\[ s_t + I_t = (1 + n)k_{t+1}. \]  
(9)

Given the initial condition \( k_0 \) and the values of \( \tau \) and \( \eta \), a competitive equilibrium for the economy is a sequence as \( \{ k_t, s_t, I_t, P_t, k_{t+1}, k_{t+2}, \ldots \} \) that satisfies equations (1)-(9) for all \( t \).

Substituting equations (2), (3) and (5)-(9) into (4) gives the following dynamic equilibrium system:
\[ \alpha^\tau Z^\eta = \frac{\theta \alpha (1 - \alpha)A}{(1 + n)[\alpha(1 + \theta)(1 + \eta) + (1 - \alpha)\eta]} \cdot \]  
(10)

where \( Z = \frac{\theta \alpha (1 - \alpha)A}{(1 + n)[\alpha(1 + \theta)(1 + \eta) + (1 - \alpha)\eta]} \cdot \) The magnitudes of the numerator and denominator are \( 10^{-3} \) and \( 10^{-4} \), respectively, because \( 0 < \theta, \alpha, \eta < 1 \). Hence, \( 0 < Z < 1 \), and the dynamic system converges to a unique, stable and nonoscillatory equilibrium.

Effects of Exogenous Variables

In the steady state, equation (10) becomes
\[ k = Z^{\eta(1-\alpha)}. \]  
(11)

The social pool benefits, individual account principal, working-period consumption and retirement-period consumption can be written as
\[ P = (1 + n)\frac{\eta}{1 + \eta}(1 - \alpha)Ak^\alpha, \]  
(12)

\[ I = \frac{\tau}{1 + \eta}(1 - \alpha)Ak^\alpha, \]  
(13)

\[ c_i = \frac{1}{1 + \eta}(1 - \alpha)Ak^\alpha - (1 + n)k, \]  
(14)

\[ c_2 = (1 + n)\left(\alpha + \eta \frac{1 - \alpha}{1 + \eta}\right)Ak^\alpha. \]  
(15)

Effect of Individual Contribution Rate

Differentiating \( k, P, I, c_1, c_2 \) and \( U \) with respect to \( \tau \) gives
\[ \frac{\partial k}{\partial \tau} = 0, \] \[ \frac{\partial I}{\partial \tau} = \frac{1}{1 + \eta}(1 - \alpha)Ak^\alpha > 0, \] \[ \frac{\partial P}{\partial \tau} = \frac{\partial c_1}{\partial \tau} = \frac{\partial c_2}{\partial \tau} = \frac{\partial U}{\partial \tau} = 0. \]

Raising the individual contribution rate induces the increase in the individual account principal, while it has no effect on the capital-labor ratio, social pool benefits, working-period consumption, retirement-period consumption and utility.

From equation (14), it can be obtained that raising the individual contribution rate increases the individual account principal. Because the voluntary savings are crowded out by one for one when the individual account principal (the mandatory savings) increases, the individual contribution rate has no effect on the capital-labor ratio, furthermore, no effect on the social pool benefits by virtue of equation (13). Similarly, it has no effect on the working-period consumption, retirement-period consumption and utility.

Effect of firm contribution rate

Differentiating \( k, P, I, c_1, c_2 \) and \( U \) with respect to \( \eta \) gives
\[ \frac{\partial k}{\partial \eta} = -k^\alpha \theta \alpha A \frac{1 - \alpha}{1 + n} \frac{1}{\alpha(1 + \theta)(1 + \eta) + (1 - \alpha)\eta} < 0, \] \[ \frac{\partial P}{\partial \eta} = \frac{(1 + n)(1 - \alpha)Ak^\alpha}{(1 + \eta)^2}, \] \[ \frac{\partial I}{\partial \eta} = \tau \frac{(1 - \alpha)Ak^\alpha}{(1 + \eta)^2} \left[ \frac{\alpha k^{-1} \partial k/(1 + \eta) - 1}{(1 + \eta) - 1} \right] < 0, \] \[ \frac{\partial c_1}{\partial \eta} = \left[ \frac{1 - \alpha}{1 + \eta} \frac{\alpha Ak^{-1} - (1 + n)\partial k/(1 + \eta)}{(1 + \eta)^{1/2}} \right] Ak^\alpha. \]
\[
\frac{\partial c_2}{\partial \eta} = (1+n)(1-\alpha)Ak^\alpha.
\]
\[
\left[ \frac{1}{(1+\eta)^2} \right] \alpha + \eta \left[ 1 - \frac{1-\alpha}{1+\eta} \right] \left( 1+\eta \right) \left( 1+n \right) \left( 1+\alpha \right) \left( 1+\eta \right) + (1-\alpha)\eta \right] \]
\[
\frac{\partial U}{\partial \eta} = \frac{1}{c_1} \frac{\partial c_1}{\partial \eta} + \theta \frac{\partial c_2}{\partial \eta}.
\]

Raising the firm contribution rate induces the decrease in the capital-labor ratio and individual account principal. The effects of the firm contribution rate on the social pool benefits, working-period consumption, retirement-period consumption and utility are dependent on the values of the related parameters.

A rise in the firm contribution rate leads to the fall in the wage, which induces the decrease in the savings and individual account principal. Thereby the capital-labor ratio falls. As for the social pool benefits, an increase in the firm contribution rate has two opposite effects: on the one hand, it directly increases the social pool benefits; on the other hand, it induces the decrease in the wage, which gives rise to the decline in the social pool benefits. Consequently, the effect of the firm contribution rate on the social pool benefits depends on the parameter values. Similarly, the effects of the firm contribution rate on the working-period consumption, retirement-period consumption and utility depend on the parameter values. Hence, we will examine the effects by simulating below.

**Effect of Population Growth Rate**

Differentiating \( k \) with respect to \( n \) gives

\[
\frac{\partial k}{\partial n} = \frac{1-\alpha}{(1+n)^2} - \frac{\partial \alpha Ak^\alpha}{\alpha (1+\theta)(1+\eta) + (1-\alpha)\eta} < 0.
\]

Partially differentiating \( P, I, c_1, c_2 \) and \( U \) with respect to \( n \) gives that most of the effects of the population growth rate on the five endogenous variables depend on the values of the related parameters. Hence, we estimate the parameters at first and then check the effects by simulating.

**Simulations**

**Estimation of Parameter Values**

According to the Chinese State Council documents, “Decision on Establishing a Unified Basic Pension System for Enterprise Employees” (State Council Document 26 in 1997) and “Decision on Improving the Basic Pension System for Enterprise Employees” (State Council Document 38 in 2005), the firm contribution rate is \( \eta = 20\% \), and the individual contribution rate \( \tau = 8\% \).

Assume that the individual discount factor per year is 0.98, which is similar to that found by Auerbach and Kotlikoff (1987) and used by Pecchenino and Utendorf (1999). One period length is usually in the interval of 25-30 years in the literature on OLG model. It is assumed to be 30 years in this model. Hence, the individual discount factor per period is \( \theta = (0.98)^{30} \).

There are several calibers for population statistics in China. Since the public pension system in urban areas is different from that in rural areas, and only the former is studied in this paper, so the caliber of “Urban Population” is selected. The population growth rate in the period from 1981 to 2011 is computed to be \( n = 2.425 \) according to the “Population and Its Composition” in China Statistical Yearbook.

The capital share of income is usually to be estimated as 0.3 in developed countries (e.g., Zhang et al., 2001; Pecchenino and Pollard, 2002; Barro and Sala-I-Martin, 2004). The labor in China is comparatively cheaper, thus, the labor share of income is lower, while the capital share of income is higher than that in developed countries. Hence, it is proper to assume that \( a \) in China is 0.35. Since what we want to see here is how the endogenous variables change relatively with the exogenous variables, the constant \( A \) can be normalized as 1. These values are baseline values of the parameters.

**Simulation on Firm Contribution Rate**

Simulating with the baseline parameter values but raising the firm contribution rate from 18% to 20% and 22%, respectively, gives the result shown in Table 1. It is shown that raising the firm contribution rate induces the decrease in the capital-labor ratio, increase in the social pool benefits, retirement-period consumption, and decrease in the individual account principal, working-period consumption and utility.

**Simulation on Population Growth Rate**

Taking the urban population from 2000 to 2011 in China Statistical Yearbook as sample, it is easy to find its trend line, \( y = 2085x + 43856, R^2 = 0.999 \) by the software of Exel. According to the trend line, one can forecast the population growth rates during 2001-2031.
and 2006-2036 which are 1.301 and 1.076, respectively. Simulating with the baseline parameter values and the forecasted population growth rates yields the result shown in Table 2. The fall in the population growth rate increases the capital-labor ratio, individual account principal, working-period consumption and utility, and decreases the social pool benefits and retirement-period consumption.

<p>| TABLE 1 EFFECT OF FIRM CONTRIBUTION RATE |</p>
<table>
<thead>
<tr>
<th>η</th>
<th>18%</th>
<th>20%</th>
<th>22%</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>0.0094</td>
<td>0.0089</td>
<td>0.0085</td>
</tr>
<tr>
<td>P</td>
<td>0.0662</td>
<td>0.0711</td>
<td>0.0757</td>
</tr>
<tr>
<td>I</td>
<td>0.0086</td>
<td>0.0083</td>
<td>0.0080</td>
</tr>
<tr>
<td>c₁</td>
<td>0.0753</td>
<td>0.0733</td>
<td>0.0713</td>
</tr>
<tr>
<td>c₂</td>
<td>0.2998</td>
<td>0.3008</td>
<td>0.3017</td>
</tr>
<tr>
<td>U</td>
<td>-3.2430</td>
<td>-3.2686</td>
<td>-3.2941</td>
</tr>
</tbody>
</table>

<p>| TABLE 2 EFFECT OF POPULATION GROWTH RATE |</p>
<table>
<thead>
<tr>
<th>n</th>
<th>2.425</th>
<th>1.301</th>
<th>1.076</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>0.0089</td>
<td>0.0164</td>
<td>0.0193</td>
</tr>
<tr>
<td>P</td>
<td>0.0711</td>
<td>0.0592</td>
<td>0.0564</td>
</tr>
<tr>
<td>I</td>
<td>0.0083</td>
<td>0.0103</td>
<td>0.0109</td>
</tr>
<tr>
<td>c₁</td>
<td>0.0733</td>
<td>0.0908</td>
<td>0.0960</td>
</tr>
<tr>
<td>c₂</td>
<td>0.3008</td>
<td>0.2504</td>
<td>0.2388</td>
</tr>
<tr>
<td>U</td>
<td>-3.2686</td>
<td>-3.1545</td>
<td>-3.1251</td>
</tr>
</tbody>
</table>

**The Social Optimum**

Since the firm contribution rate as a policy variable has effect on the capital-labor ratio, it is possible to find the optimal firm contribution rate to maximize the social welfare. The social welfare is defined as the sum of the lifetime utilities of all current and future generations (Blanchard and Fischer, 1989 and Groezen et al., 2003 also use an analogous social welfare function):

\[
W = \theta n c_{20} + \sum_{i=0}^{\infty} \rho^i (\ln c_{1i} + \theta \ln c_{2i+1} ).
\]  

(16)

where \( \rho \in (0,1) \) is the social discount factor, which reflects the preference of the social planner. The resource constraint is

\[
k_i + Ak_i^\nu = (1+n)k_{i+1} + c_{1i} + c_{2i}/(1+n).
\]  

(17)

The initial condition is that \( k_0 \) is given, and the terminal condition is \( k_\infty = 0 \).

The social planner maximizes the social welfare subject to the resource constraint, initial condition and terminal condition. Appendix A gives the first-order conditions for the social welfare maximization problem:

\[
\theta(1+n)c_1^* = \rho c_2^* ,
\]

(18)

\[
k^* = \left( \frac{1+n-\rho}{\rho \alpha A} \right)^{1/(\alpha-1)}.
\]

(19)

where the superscript * denotes the optimal steady state values of variables. The capital-labor ratio satisfying equation (19) is at the modified golden rule level, which means that the social welfare reaches the maximum.

In order to maximize the social welfare of the decentralized economy in the steady state, we control the policy variable to adjust the capital-labor ratio of the decentralized economy in the steady state to the modified golden rule level, namely, \( k = k^* \). Substituting equation (19) into equation (10) and rearranging gives

\[
\eta^* = \theta(1-\alpha)(1+n-\rho) - \rho \alpha(1+n)(1+\theta) \left( \frac{\rho}{\rho(1+n)(1+\theta \alpha)} \right).
\]  

(20)

The optimal firm contribution rate depends on the individual discount factor \( \theta \), social discount factor \( \rho \), capital share of income \( \alpha \), and population growth rate \( n \).

Differentiating \( \eta^* \) with respect to \( n \) can give the effect of the population growth rate on the optimal firm contribution rate. It is shown that the effect is dependent on the relevant parameter values, which also can be checked by simulation.

It is necessary to estimate the social discount factor at first, which indicates how much the government weights different generations in its social welfare calculations. Hence, it should be estimated according to the government’s regulations. Based on the Chinese State Council Document 38 in 2005, one can get that the optimal firm contribution rate adopted by the government is 20%. Substituting the relevant baseline values into equation (20) and calculating repeatedly until the equation holds, one can get that \( \rho = 0.4017 \).

Simulating with the forecasted population growth rates and the baseline values of \( \theta \), \( \alpha \) and \( \rho \) gives the result shown in Table 3. The optimal firm contribution rate falls in the population growth rate. This is contradictory to the general image. The fall in the population growth rate increases the capital-labor ratio and furthermore the wage. When the effect of the
increase in the two variables dominates that of the fall in the population growth rate, the income in retirement-period rises. The social optimum needs to retain the optimal retirement-period consumption. Hence, the optimal firm contribution rate has to be adjusted to fall with the population growth rate.

Table 3 Optimal firm contribution rates under different population growth rates

| n   | η*  
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>2.425</td>
<td>20.0%</td>
</tr>
<tr>
<td>1.301</td>
<td>15.8%</td>
</tr>
<tr>
<td>1.076</td>
<td>14.4%</td>
</tr>
</tbody>
</table>

Conclusions

Employing the OLG model with general equilibrium, this paper studies the partially funded public pension system in urban areas of China, and examines the effects of the individual contribution rate, firm contribution rate and population growth rate on the capital-labor ratio, social pool benefits, individual account principal, consumption and utility, and finds the optimal firm contribution rate. Adopting logarithmic utility function and Cobb-Douglas production function, we use the data in China such as the urban population, capital share of income, public pension policy variables, etc., to simulate the effects of the exogenous variables on the endogenous variables and the change in the optimal firm contribution rate when the population growth rate falls.

The main results are as follows. Raising the individual contribution rate increases the individual account principal, whereas the rate has no effect on the capital-labor ratio, social pool benefits, consumption and utility. Raising the firm contribution rate increases the social pool benefits and retirement-period consumption, whereas decreases the capital-labor ratio, individual account principal, working-period consumption and utility. The fall in the population growth rate induces the rise in the capital-labor ratio, individual account principal, working-period consumption and utility, whereas the fall in the social pool benefits and retirement-period consumption. The optimal firm contribution rate depends on the individual discount factor, social discount factor, capital share of income and population growth rate. It decreases along with the population growth rate.

The above results have the following policy implications: (a) In order to retain investment to keep a rational economic growth in China, it is necessary to reduce the firm contribution rate or strictly implement the population policy. (b) To increase the social pool benefits, it is necessary to raise the firm contribution rate or properly relax the population policy. (c) To increase the individual account principal, it is necessary to raise the individual contribution rate or reduce the firm contribution rate or strictly implement the population policy. (d) To increase the consumption of the workers, it is necessary to reduce the firm contribution rate or strictly implement the population policy. (e) To increase the consumption of the retirees, it is necessary to raise the firm contribution rate or properly relax the population policy. (f) To raise the utility level, it is necessary to reduce the firm contribution rate or strictly implement the population policy.

Integrating the above six aims, it will do more good than harm to raise the individual contribution rate, reduce the firm contribution rate and strictly implement the population policy. The following two results strengthen the implication of reducing properly the firm contribution rate. Firstly, raising the firm contribution rate increases the social pool benefits and consumption of the retirees. If the interim aim is to increase the social pool benefits and consumption of the retirees and the ultimate aim is to increase the utility, then raising the firm contribution rate can realize the median aim but runs counter to the ultimate aim. Secondly, the optimal firm contribution rate decreases with the population growth rate. This is derived from the social welfare maximization and widely different from general image.

Appendix A

The Lagrange function for the social welfare maximization problem is

\[
L = \cdots + \rho^{t-1}(\ln c_{t-1}^{1} + \theta \ln c_{t-1}^{2}) + \lambda_{t-1}\left[k_{t-1}^{1} + Ak_{t-1}^{2} - (1+n)k_{t-1} - c_{t-1}^{1} - c_{t-1}^{2} + \frac{C_{t-1}^{1}}{1+n}\right] + \rho^{t}(\ln c_{t}^{1} + \theta \ln c_{t}^{2}) + \lambda_{t}\left[k_{t}^{1} + Ak_{t}^{2} - (1+n)k_{t} - c_{t}^{1} - c_{t}^{2} + \frac{C_{t}^{1}}{1+n}\right] + \rho^{-t+1}(\ln c_{t+1}^{1} + \theta \ln c_{t+1}^{2}) + \lambda_{t+1}\left[k_{t+1}^{1} + Ak_{t+1}^{2} - (1+n)k_{t+1} - c_{t+1}^{1} - c_{t+1}^{2} + \frac{C_{t+1}^{1}}{1+n}\right] + \cdots
\]

where \(\lambda_{t}\) is the Lagrange multiplier for the resource constraint in period \(t\). Let the partial derivatives of \(L\) with respect to \(c_{t}, c_{t}^{2}\) and \(k_{t}\) be zero, and rearranging them at the steady state \((k^{*}, c_{t}^{*}, c_{t}^{2})\) gives the first-order
conditions for the social welfare maximization problem.

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