A Review on Stability and Vibration Control of Piezolaminated Composite/FGM Plate Structures

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ABSTRACT
Piezoelectric materials have gained substantial attention due to their potential applications as sensors and actuators for controlling the response of structures. Using the direct and converse piezoelectric effects, surface bonded piezoelectric sensor and actuator layers may be employed to adequately control the deflection, vibrations, shape and buckling of the structures. In response to remarkable increase in research activities in stability and vibration control of composite/functionally graded material (FGM) plates using piezolaminates in last few years, this review paper summarizes the research progress in following categories: stability analysis of piezolaminated composite plates and piezolaminated FGM plates, vibration control of piezolaminated composite plates and piezolaminated FGM plates.

1. INTRODUCTION

During the past decade, smart materials and structures or intelligent material systems have captured the attention of many engineering professionals and academics because of their potential application as sensors and actuators for controlling the response of structures. Smart structure consists of multifunctional components which activates on sensing and carry out an appropriate response in a controlled and timely manner. It has the capability to respond to a varying external environment as well as internal environmental conditions. Smart structural components are used in aerospace structures, high speed aircrafts and structures where high thermal environment induces buckling instability and vibrations in the structures. The detailed information about piezo history, piezo sensors, actuators and terminology related to piezo analysis is given by [1,2,3]. For various purposes smart materials are used in the realization of smart structures and focused on the analysis of smart composite structures to improve their performance. The constitutive behavior of these material couples their mechanical response i.e. stress and strain with other physical fields, this way a mechanical stimulus can be transformed into a non-mechanical effect and conversely, a non-mechanical stimulus can be converted into a mechanical effect. This is what happens in piezoelectric materials, where the electric behavior is coupled with the mechanical one due to the direct and converse piezoelectric effect. This effect enables the material to behave as an actuator or a sensor. The smart composite structures are made up of composition of different type of material laminates such as metal, ceramic, fiber reinforcement etc. but such types of composite structures are undergoing delamination because of abrupt changes in material properties, extreme environmental conditions and weakness of interfaces of layers placed between two adjacent laminates of composite structures. In the manufacturing process of composites, interlaminar bonding may be weaker due to the introduction of small flaws or micro cracks under various loading conditions.

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during the service time and this may lead to inter-laminar instability, but FGM can be used to overcome this problem. Koizumi [4] highlighted the FGM activities in Japan. The concept of FGMs was proposed in 1984 by materials scientists in the Sendai area as a means of preparing thermal barrier materials. Continuous changes in the composition, microstructure, porosity, etc. of these materials results in gradients in such properties as mechanical strength and thermal conductivity.

The main objective of the paper is to carry out the review on stability analysis and vibration control of piezolaminated composite and FGM plates and provide the current research activities. It is intended to help readers and researchers to apprise current progress in stability as well as vibration control analysis of piezolaminated composite plates/FGM plates.

2. REVIEW ON STABILITY ANALYSIS OF PIEZOLAMINATED COMPOSITE PLATES/FGM PLATES

Thin and light weight structures and material suffers by buckling and dynamic instability and the research is focused to improve their performance using smart materials. The following section highlights the review on buckling and post-buckling analysis of piezolaminated composite plate and piezolaminated FGM plate.

2.1. Stability Analysis of Piezolaminated Composite Plates

Chandrashekhara and Bhatia [5] developed a finite element model to study the dynamic buckling behavior for composite plates with integrated sensors and actuators. The finite element model is based on the first order shear deformation plate theory (FSDT) in conjunction with linear piezoelectric theory. Icardi and Di Sciuva [6] investigation is carried out to study the 3D stress field of multilayered intelligent plate based on the von Karman strain-displacement relations and developed a third order zigzag layerewise plate theory for multilayered, intelligent, anisotropic plates with a surface-bonded piezoelectric actuator layer. The numerical study has been carried out on simply supported cross ply plates with top and bottom actuators in cylindrical bending under distributed transverse loading. Oh et al. [7] formulated non-linear finite element equations based on the layerwise plate theory for a piezolaminated plate subject to thermal and piezoelectric loads and carried out postbuckling analysis as well as vibration analysis considering large thermo-piezoelectric deflections for a square composite plate with fully and partially bonded piezoelectric actuators. Shen [8] presented postbuckling analysis for a simply supported shear deformable laminated plates with piezoelectric actuator subjected to the combined action of mechanical, electrical and thermal loading. The governing equations of the laminated plate theory based on Reddy’s higher order shear deformation theory which includes thermo-piezoelectric effect and a perturbation technique is used to determine postbuckling loads and equilibrium path. Shen [9] presented thermal postbuckling analysis for a simply supported shear deformable laminated plates with piezoelectric actuator subjected to the combined action of thermal and electrical loading. A parabolic distribution of temperature field is assumed over the plate surface and a uniform distribution in the thickness direction. A mixed Galerkin-perturbation technique is employed to determine thermal buckling loads and postbuckling equilibrium paths.

Varelis and Saravas [10] developed a finite-element formulation to predict the initial buckling of smart piezoelectric plate structures with piezoelectric actuators and sensors. The buckling analysis is carried out on piezoelectric plates subjected to various combinations of applied electric potentials and in-plane forces. Varelis et al. [11] presented the theoretical framework for coupled buckling and post buckling analysis of piezoelectric adaptive plate structures. The formulation is based on coupled mixed field piezoelectric laminated theory with consideration of stress stiffening and geometric effect due to large rotations to study the different cases such as plate with different positioned actuator patches, hinged plate with asymmetric actuator, electromechanical buckling and active buckling of plate. Wang et al. [12] studied the dynamic stability analysis of active vibration control of piezoelectric composite plates using the negative velocity feedback control law. The analysis is carried out by using Lyapunov’s energy functional which is based on derived general governing equations of motion by assuming active damping. The dynamic stability of plate embedded with piezoelectric layers on the top and the bottom surface as actuator and the sensor layers respectively is investigated by Kim and Kim [13] with the active damping layer under a thrust using FSDT and finite element method. Kapuria and Achary [15] presented an exact 3D piezothermoelasticity solution for buckling of simply-supported symmetrically laminated hybrid cross-ply plates with surface bonded or embedded piezoelectric layers using state space approach. The buckling is considered under uniform temperature with the piezoelectric layers under charge free open circuit condition as well as closed circuit condition with zero and non-zero actuation potentials. Kapuria and Achary [16] extended previous work of Kapuria [14] to present an efficient coupled geometrically nonlinear zigzag theory for hybrid plates including geometric nonlinearity due to deflection only in the sense of Von Karman. Kapuria and Achary [17] developed efficient coupled geometrically nonlinear zigzag theory for hybrid plates under electro-thermo-mechanical load and used to obtain the thermal buckling response of symmetrically laminated hybrid plates. Akhras and Li [18] proposed a new finite layer method for three-
dimensional static, vibration and stability analysis of piezoelectric composite plates. Numerical studies presented to verify the performance of the proposed method and effects of electrical conditions, side-to-thickness ratio and number of plies. Giannopoulos et al. [19] presented buckling of smart beams and plates under complex loading in combination with discrete layer kinematic assumptions for through the thickness behavior of the structure. The thermal, electrical and mechanical coupling formulation is presented and incorporated in a finite element solver using discrete layer kinematics and quadratic finite element. In above scenario, there is thermal, mechanical, electrical buckling and postbuckling analysis is presented with different parametric studies. But in composite structures, interlaminar bonding imperfection has received some attention because in the manufacturing process, interlaminar bonding may be weaker due to the introduction of small flaws or micro-cracks are induced under various conditions during the service time.

Kim and Lee [20] investigated the buckling of orthotropic rectangular laminates with weak interface. The state-space formulation is directly established from three dimensional theory of elasticity and spring layer model is evaluated. Akhras and Li [21] studied the three-dimensional thermal buckling analysis of symmetrical cross-ply piezoelectric composite plates with full coupling between the thermal, electrical and mechanical fields using finite layer method proposed by Akhras and Li [18]. Investigation is carried out to show the effects of side-to-thickness ratio, number of plies, electrical conditions, thermal-electrical coupling and heat conduction during the buckling process on the thermal buckling behaviors of piezoelectric laminates. Kim and Lee [22] extended previous work of Kim and Lee [20] to investigate the buckling of an orthotropic piezoelectric rectangular laminate with weak interfaces. The numerical results investigate the interfacial damage as well as it shows that the sensitivities of buckling stress parameter to the interfacial damage depend on various factors. Yang and Huang [23] presented the dynamic stability analysis of a simply supported 3D braided composite laminated plate with surface bonded piezoelectric layers, under the influence of electrical and periodic in-plane mechanical loads. Shariyat [24] presented the dynamic buckling analysis by considering the coupling of mechanical, thermal and electrical field. Here investigation is carried out to show the effect of thermo piezoelectricity on the dynamic buckling for suddenly applied thermal and mechanical loads for piezolaminated composite plates. Formulation is based on higher order shear deformation theory (HSDT) by considering initial geometric imperfection and temperature dependency of the material. Akhras and Li [25] extended previous work of Akhras and Li [21] to the 3D thermal buckling analysis of simply supported rectangular piezoelectric antisymmetric angle-ply laminates and investigated to show the effect of number of layer on thermal buckling of square laminates and adiabatic thermal buckling of square (p/45°/−45°/45°/−45°/p) laminates, square antisymmetric angle ply laminates, square (p/0°/90°/90°/0°/p), (p/45°/−45°/45°/−45°/p), and rectangular (p/45°/−45°/45°/−45°/p) piezoelectric laminates. Akhras and Li [26] extended the finite layer method to study a 3D stability analysis of simply supported rectangular piezoelectric antisymmetric angle-ply laminates. The Lagrangian polynomials are employed for interpolations of all the generalized displacement components in the thickness direction.

Pradyumna and Gupta [27] studied the dynamic stability behavior of laminated composite plates with piezoelectric layers subjected to periodic in-plane load. The analysis is based on finite element method and earlier developed modified first order shear deformation theory (MFSDT) by Tanov and Tabiei [28]. The formulation includes the effects of transverse shear, in-plane, and rotary inertia and Bolotin’s approach is used to obtain the boundaries of dynamic instability regions. Shen and Zhu [29] studied compressive postbuckling and thermal postbuckling under thermal environments and uniform temperature rise for a shear deformable laminated plate with piezoelectric fiber reinforced composite actuators based on a HSDT including thermo-piezoelectric effects. The compressive and thermal postbuckling behaviors of perfect, imperfect, symmetric cross-ply and antisymmetric angle-ply laminated plates with fully covered actuators under the influence of different sets of thermal and electric loading conditions. Moghadam et al. [30] developed an analytical model for bending analysis of rectangular hybrid general cross-ply piezolaminated plates with any arbitrary clamped/simply supported boundary conditions under thermo-electro-mechanical loading. Jabbari et al. [31] evaluated the buckling load of porous circular plates integrated with piezoelectric actuator layers, subjected to uniform in-plane radial compression and applied constant voltage on piezo layers. The effects of piezoelectric layers on the buckling load, piezoelectric layer-to-porous plate thickness ratio and variation of porosity are presented in this paper. The extension of this study made in Khoshshidvand [32] by integrating plate with piezoelectric sensor and actuator patches, subjected to uniform in-plane radial compression and studied the effects of piezoelectric layers on feedback gain. Wankhade and Bajoria [33] studied stability of piezolaminated plates subjected to combined action of electrical and mechanical loading. Wankhade and Bajoria [34] studied buckling analysis of piezolaminated plate subjected to combined action of electro-mechanical loading using HSDT and finite element method. Piezolaminated plates consist of cross ply symmetric and antisymmetric orientation of laminates with piezolayer attached at the top and bottom of the plate.

2.2. Stability Analysis of Piezolaminated FGM Plates

Liew et al. [35] investigated the buckling and postbuckling behavior of FGM hybrid plate which is surface bonded
with piezoelectric actuators under combined influence of uniform temperature change, in-plane forces and constant applied control voltage. Hybrid plate is formed by combining ceramics and metals i.e. zirconia and aluminum. Modeling is based on Reddy’s higher order shear deformation theory by applying semi analytical one dimensional differential quadrature (DQ) approximation based iterative approach which is used to determine the postbuckling response of FGM plate and Galerkin procedure is used to evaluate non-linear algebraic equation, further which is useful for determining postbuckling path by using iterative approach. Yang et al. [36] investigated the non-linear bending response of shear deformable FGM rectangular plate which is mixture of zirconia and aluminum, covered with piezoelectric actuator layers under application of thermo-electro-mechanical loads. Material properties are assumed to be graded in thickness direction according to power-law distribution and also it is independent of temperature and electrical field. Formulation is based on Reddy’s higher order shear deformation theory and von Karman assumption for influence of geometric non-linearity. Shen [38] studied the postbuckling analysis for a simply supported, shear deformable functionally graded plate with piezoelectric actuators by extending the previous work of Shen [8,37] to the case of mid-plane symmetric FGM hybrid plates subjected to the combined action of mechanical, electrical and thermal loads. Two sets of material mixture for FGMs are considered i.e. silicon nitride and stainless steel and the other is zirconium oxide and titanium alloy. The temperature field considered is assumed to be of uniform distribution over the plate surface and through the plate thickness and the electric field considered only has non zero valued component $E_Z$. Reddy’s higher order shear deformation plate theory is used in the analysis and a two step perturbation technique is employed to determine buckling loads and postbuckling equilibrium paths. Shen H.S. [39] studied the thermal postbuckling analysis for a simply supported, shear deformable functionally graded plate considering the heat conduction and temperature-dependent material properties and extended previous work of Shen [37, 38] to the case of geometrically mid-plane symmetric FGM plates subjected to thermal loads. A two-step perturbation technique is employed to determine buckling temperature and postbuckling equilibrium paths.

Chen et al. [40] presented the buckling and postbuckling analysis of piezoelectric FGM plate subjected to non-uniformly distributed loads, heat and voltage by using element free Galerkin method by employing penalty coefficients. First calculation is made for pre-buckling stresses of plates subjected to non-uniformly distributed loads and then calculated the buckling load and temperature of the plate by considering Mindlin plate assumption. For validation of above analysis some numerical studies are carried out such as, analysis of rectangular piezoelectric FGM plate which is subjected to pair of in-plane concentrated loads and square piezoelectric FGM plate subjected different loading such as, several pairs of partial uniform in-plane edge loads, one axial load and two shear loads. Results based on above study show that if small amount of voltage increases then buckling parameter decreases. Shen [41] extended previous work of Shen [38] to investigate the nonlinear compressive postbuckling under thermal environments and thermal postbuckling due to a uniform temperature rise are presented for a simply supported, shear deformable FG plate which is mixture of silicon nitride and stainless steel with piezoelectric fiber reinforced composite actuators. The analysis is based on a HSDT with von Karman type of kinematic nonlinearity.

Shariyat [42] presented the vibration and dynamic buckling of FGM rectangular plates which is mixture of aluminum oxide and titanium alloy with surface bonded piezoelectric sensors and actuators under the influence of thermo-electro-mechanical loading. Finite element formulation is based on higher order shear deformation theory. Dynamic buckling is evaluated for plate is already subjected to electric potential and thermal and electrical loads under suddenly applied mechanical compression. The minus voltage is applied then thermal and mechanical buckling loads are having slightly higher value in case of thin plate, but same result is not acceptable for thick plates. Ke et al. [43] presented the analytical solutions for the flexural vibration and buckling of beams made of FGM containing open edge cracks based on Timoshenko beam theory and the rotational spring model. Mirzavand and Esfami [44] presented the thermal buckling of FG rectangular plates which is mixture of zirconium oxide and titanium alloy integrated with surface-bonded piezoelectric actuators. The third order shear deformation plate theory is employed to account for the transverse shear strains. The temperature dependent of the material properties are considered. The buckling analysis of the plate under thermal loadings is carried out using the Reitz method. They indicate that the buckling temperature difference can be controlled by applying a suitable voltage on the actuator layers.

Jadhav and Bajoria [45] and Bajoria and Jadhav [46] presented the stability analysis of a piezolaminated metal based FG plate subjected to electrical and mechanical loading. The finite element model is derived with the von Karman hypothesis and degenerate shell element using the FSDT and HSDT for thin and thick FGM plate respectively. The analysis is carried out on a newly introduced metal based FGM material which is a mixture of aluminum and stainless steel. Khorshidvand et al. [47] studied the thermal buckling of circular plates made of FGMs with surface bonded piezoelectric layers subjected to applied constant voltage, uniform temperature rise, nonlinear and linear temperature variation through the thickness for immovable clamped edge of boundary conditions. The general thermoelastic nonlinear equilibrium and linear stability equations for the piezoelectric FG plate are derived using the variational formulations. Panda and Sopan [48] developed finite element model for geometrically non-
3. REVIEW ON VIBRATION CONTROL OF PIEZOLAMINATED COMPOSITE PLATES/FGM PLATES

Vibrations of the piezolaminated structures can be effectively controlled using the direct and converse piezoelectric effects with distributed sensors and actuators. Sensor detects the oscillations and the actuator controls the vibration of the system. The classical control laws are constant gain negative velocity feedback and Lyapunov’s feedback which are based on output feedback. In the case of classical control laws, the gains are arbitrarily chosen, whereas in case of optimal control law, an optimal control gain is obtained which minimizes as object functions. Several studies have been carried out for vibration control of piezolaminated plate and piezolaminated FGM plate structures. Review is presented to highlight the vibration control studies of the composite / FGM plate structure using piezoelectric effect in following section.

3.1. Vibration Control of Piezolaminated Composite Plates

Lawson [49] investigated the characteristic frequencies of infinite piezoelectric plates vibrating between the grounded electrodes of a plane parallel condenser. The investigation exhibits that the frequencies depend on the piezoelectric constants as well as the elastic constants of the crystal. Tiersten [50] investigated the thickness vibrations of an infinite anisotropic plate with electrodes coated on both surfaces are investigated using the linear piezoelectric equations. In the general case, the three fundamental solutions of the differential equations were found to couple at the traction free surfaces of the plate. Hruska [51] investigated the similar problem of Lawson [49] to study an infinite piezoelectric plate vibrating between two electrodes of a plane parallel condenser for the case of a small d.c. potential maintained across the condenser electrodes. Chandrashekhar and Agarwal [52] developed a finite element formulation for laminated composite plates with integrated piezoelectric sensors and actuators using FSDT. The developed mathematical model is similar to that of Lee [53] however; the FSDT was used instead of classical laminated plate theory (CLPT). The improved structural behavior is demonstrated using a constant gain feedback control. Hwang et al. [54] investigated the combined effects of passive and active control on the vibration control of a composite laminated plate with piezoelectric sensors/actuators using finite element formulation and modal analysis. The equations of motion of the system were formulated using CLPT with the induced strain actuation and Hamilton’s principle. The total charge developed on the sensor layer is calculated from the direct piezoelectric equation.

Tzou and Fu [55] investigated the active vibration controls of the plate with various sizes of sensors/actuators and control algorithm, the proportional feedback and Lyapunov control were presented and carried out the free vibration analysis to find out natural frequencies and mode shapes. Chandrashekhara and Tenneti [56] developed a finite element model for the active control of thermally induced vibration of laminated composite plates with piezoelectric sensors and actuators. The direct and converse piezoelectric effects are coupled with a constant gain feedback control algorithm to actively control the dynamic response of the plate in a closed loop. A C\(^0\) continuous nine noded shear flexible element is implemented to model the plate. Baz and Ro [57] presented a finite element analysis of the dynamic control of the flat plates which are partially treated with patches of the active constrained layer damping (ACLD) treatment and demonstrated its ability to control the bending vibration of flexible plates. Batra and Liang [58] studied the steady state vibrations of a simply supported rectangular laminated elastic plate with embedded piezoelectric actuators and sensors by using the three-dimensional elasticity theory. Han et al. [59] presented an experimental study for active vibration control of composite beams and plates and developed an analytical model using Ritz method to calculate the sensor output history and control voltage history for the composite beam with piezoelectric sensors and actuators. The results of an analytical model are validated with experimental investigations. Abramovich and Meyer [60] presented an exact elasticity solution for forced induced vibrations of a piezolaminated elastic beam being driven by time harmonic voltages applied to the actuators on top and bottom surfaces. The analysis is based on Fourier series.

Goswami and Kant [61] presented a generalised finite element formulation of a consistent plate model for active vibration control of stiffened laminates integrated with piezoelectric polymer layers acting as distributed sensors and actuators. Total charge developed on the sensor layer is calculated from the direct piezoelectric equations. Kang et al. [62] analyzed multi-modal vibration control of the cantilevered laminated composite plate using collocated piezoceramic sensors/actuators and verified experimentally for various fiber orientations. The research highlighted an analytical approach to evaluate the passive and the active vibration control of the plate. Lam and Ng [63] presented the theoretical formulations using CLPT and Navier solutions for the analysis of the laminated composite plates with integrated sensors and actuators subjected to both mechanical and electrical loadings. An active control of the dynamic response of the integrated plate structures through closed loop control is carried out using negative force-cum-moment
feedback control algorithm which is coupling with the direct and converse piezoelectric effects. Lin and Huang [64] investigated the vibration control of beam-plates with bonded piezoelectric sensors and actuators based on finite element method and presented the basic equations for piezoelectric sensors and actuators. The equation of motion is derived for a beam-plate structure bonded with pairs of piezoelectric sensors or actuators by using the Hamilton's principle.

Liu et al. [65] developed finite element model for the shape control and active vibration suppression of laminated composite plates with integrated piezoelectric sensors and actuators using CLPT and the principle of virtual displacements. Numerical investigations show the influence of stacking sequence and position of sensors and actuators on the response of the plate. Reddy [66] presented a general formulation for laminated composite plates with piezoelectric actuators and sensors. The formulation is based on CLPT, FSDT and third order shear deformation theories incorporating the thermo-electro-mechanical coupling, von Karman type geometric nonlinearity and time dependency. Saravanos [67] presented a coupled electromechanical theory for composite laminates with multiple piezoelectric layers connected to passive electric circuits and a Ritz solution is used for predicting the modal damping, modal frequencies and damped response of composite piezoelectric plates. The equations of motion for passive laminate systems were solved for the case of simply supported plates and eigenvalues of the damped plate were calculated to obtain the modal frequencies and damping. Balamurugan and Narayanan [68] developed a new piezolaminated quadrilateral composite plate/shell finite element and applied to a composite cantilevered plate and cantilevered semicircular shell with distributed PZT piezoceramic sensor and actuator on the top and bottom surfaces. The classical control methods for different kinds of loading environments are used to control vibration composite laminated plates. Results predicts that the velocity feedbacks like constant gain negative velocity feedback and Lyapunov feedback are more effective in controlling the vibrations when compared to displacement feedback like direct proportional feedback but LQR control methods are more effective in controlling the vibration with lesser peak voltages. Valoor et al. [69] developed a neural network based control system for self adapting vibration control of laminated plates with piezoelectric sensors and actuators. The finite element model of the plate with integrated piezoelectric sensor and actuators is used to simulate the vibration of the plate.

Kang et al. [70] investigated the interaction between active and passive vibration control characteristics by numerically and verified experimentally. The finite element method is used for the analysis of dynamic characteristics of the laminated composite beams. Experiments on the active vibration control of the laminated composite beams were carried out using velocity feedback control. Mukherjee et al. [71] presented active control of stiffened composite plate using piezoelectric materials. The stiffener had been formulated such that, it could have any shape in plan and need not pass through the nodal lines of the finite element mesh. The established formulation could be effectively employed in the analysis of plates with eccentric stiffener. Gao and Shen [72] developed an incremental finite element equations considering the geometrical non-linearity of structures with piezoelectric patches based on virtual velocity incremental variational principles with the assumption of weak mechanical and electric coupling. The numerical investigation exhibits that piezoelectric actuators can produce significant damping and control transient vibration effectively and the numbers and locations of the piezoelectric actuators have influence on the shape control and the vibration control of the structures.

Kulkarni and Bajoria [73,74] presented a new finite element model for the piezolaminated plates/shells using the HSDT and studied the active control of piezolaminated composite plates/shells. Narayanan and Balamurugan [75] presented finite element formulation for active vibration control of shear deformable piezolaminated beam, plate and shell including the stiffness, mass and electromechanical coupling effects of distributed piezoelectric sensor and actuator layers. For the case of plate/shell elements the thermo electromechanical coupling is considered and studied the active vibration control using the classical control methods like constant gain negative velocity feedback and Lyapunov feedback. Quek et al. [76] investigated the placement of collocated piezoelectric sensor/actuator pairs in a laminated composite plate using the finite element method. Two performance functions based on the modal controllability and system controllability are proposed as indices for damping out the free vibration. Chen et al. [77] investigated the bending and free vibration of an imperfectly bonded orthotropic piezoelectric rectangular laminates using a three dimensional state-space approach. The interlaminar bonding of the host elastic laminate was assumed to be imperfect, described by a spring layer model while the bonding between the host elastic laminate and the surface piezoelectric actuator and sensor layers was perfect. A general spring layer was adopted to model the bonding imperfections. Chen et al. [78] employed the same three dimensional state-space approach to investigate the static and dynamic problems of simply supported adaptive angle ply laminates in cylindrical bending featuring interlaminar bonding imperfections. Liew et al. [79] developed an element free Galerkin method based on the FSDT for the shape control and vibration suppression of the piezolaminated composite plates. Also investigation has been carried out on the free vibration analysis of simply supported square piezolaminated plates by considering different orientations of composite substrates and locations of piezoelectric patches.

Moita et al. [80] presented a finite element formulation for active vibration control of thin plate laminated structures
with integrated piezoelectric layers acting as sensors and actuators. The model is based on the Kirchhoff’s CLPT and can be applied to plate and shell adaptive structures. Newmark method is considered to calculate the dynamic response of the laminated structures. Raja et al. [81] studied the influence of active stiffening on the frequency control of piezo-hygro-thermo-elastic laminated plates and shells for various elastic modes using coupled piezoelectric finite element formulation involving a hygrothermal strain field based on virtual work principles. Vel et al. [82] presented an analytical solution for the cylindrical bending vibrations of linear piezoelectric laminated plates which is obtained by extending the Stroh formalism to the generalized plane strain vibrations of piezoelectric materials. Tzou et al. [83] presented a thorough review on smart materials including material histories characteristics, material varieties, limitations, sensor/actuator/structure applications of piezoelectric, shape memory materials, electro and magnetostrictive materials, and magnetorheological fluids, polyelectrolyte gels, superconductors, pyroelectrics, photostrictive materials, photothermoelectrics, magneto-optical materials. Duan et al. [84] presented the free vibration analysis of piezoelectric coupled annular plates using the Kirchhoff’s and Mindlin plate models. Sinusoidal function is used to describe the distribution of electric potential along the thickness direction of thin and thick plate. Maxwell’s static electricity equation is included as one of the governing equations. The results are verified against three dimensional finite element analysis using ABAQUS.

Huang and Shen [85] studied the nonlinear vibration and dynamic response of simply supported shear deformable cross ply laminated plates with piezoelectric actuators subjected to mechanical, electrical and thermal loads. The theoretical formulation is based on the HSDT and general von Karman’s type strains including thermo-piezoelectric effects. Oh [86] studied the nonlinear dynamics of active piezolaminated plates to investigate thermo-piezoelectric snap-through phenomena. A multi-field layer wise finite element is proposed for high accuracy and nonlinearity of displacement, electric and thermal fields. The thermoelastic postbuckling of the structural models is investigated and the characteristics of piezoelectric active responses are studied for finding snap-through piezoelectric potentials and the load-path tracking map. Peng et al. [87] developed a methodology for piezoelectric patches placement optimization and introduces adaptive feedforward control into smart structure vibration control. The optimization methodology is based on modeling using ANSYS. Du et al. [88] obtained exact solutions for thickness vibrations of a piezoelectric plate under uniform biasing acceleration with the consideration of the piezoelectric stiffening.

Chen et al. [89] extended previous work of Chen et al. [77] to investigate bending and free vibrations of simply supported cross ply piezolaminated cylindrical panel featuring with interlaminar bonding imperfections. Qu et al. [90] investigated the vibration behavior of a piezoelectric composite plate with cracks and developed the dynamic model based on the principle of minimum energy and analyzed the effects of cracks and piezoelectric materials on mode shapes. Kulkarni and Bajoria [91] extended previous work of Kulkarni and Bajoria [73] to investigate the large deformation analysis of piezolaminated smart structures using finite element analysis based on FSDT and HSDT. Heidary and Eslami [92] studied the piezo-control of forced vibrations of the laminated thermoelastic plate and presented the dynamic response under the rapidly applied mechanical excitation and prescribed thermal loading. The structural vibrations induced in a laminated thermoelastic plate are controlled by appropriate control voltage applied to piezoelectric layers. Ghasemi-Nejad et al. [93] presented study on the use of piezoelectric stack and monolithic patch actuators in finite element analysis and examined the effects of the actuator location on the vibration suppression and the level of optimum voltage. Balamurugan et al. [94] studied the active vibration control performance of the piezolaminated smart composite plates using finite element model based on HSDT by applying various control strategies.

Kulkarni and Bajoria [95] presented the geometrically nonlinear finite element analysis of smart structures and presented numerical investigations of piezolaminated composite beams, plates and shell structures using the FSDT and HSDT. Lee and Guo [96] developed a finite element simulation model for the active control of nonlinear composite panel vibration using von Karman nonlinear strain-displacement relations for large deflection responses and linear piezoelectric constitutive relations under random excitation. Four control methods, velocity feedback, lead, lag and $H\infty$ are employed and examined for cases of small and large amplitude vibrations. The $H\infty$ method works better under small excitations with small piezoelectric actuators and the lag compensator performs better under large excitations with large piezoelectric actuators. Della and Shu [97] developed a mathematical model for the vibration of beams with piezoelectric inclusions. The piezoelectric inclusion in a non-piezoelectric matrix is analyzed as two inhomogeneous inclusion problems as elastic and dielectric by using Eshelby’s equivalent inclusion method. The Euler-Bernoulli beam theory and Rayleigh-Ritz approximation technique are used for analysis. In addition a parametric study was conducted to investigate the influence of the energies due to piezoelectric coupling on the natural frequency of the beam.

Zhang and Shen [98] presented an analytical formulation for structural vibration control of laminated plates consisting of piezoelectric fiber reinforced composite layers and orthotropic composite layers. The active controlled electric field was applied to the piezocomposite layers equipped with Interdigitated Electrodes (IDE). Based on the thin plate theory the governing differential equations for axial vibra-
tion and transverse vibration are established. The solution is obtained through the separation of variables and Fourier expansion method. Pietrzakowski [99] formulated models of piezoelectric coupled laminated plates based on Kirchhoff’s and Mindlin’s kinematic assumptions involving the electric potential distribution, which satisfies the Maxwell electrostatics equation. In the first model the displacement field is based on the Kirchhoff hypothesis and in second model the Mindlin plate theory is applied. Dash and Singh [100] studied the nonlinear free vibration of geometrically nonlinear shear deformable laminated plate with embedded and/or surface bonded piezoelectric layers in Green Lagrange sense. The formulation is based on the HSDT. Jayakumar et al. [101] studied nonlinear free vibrations of simply supported piezolaminated rectangular plates with immovable edges based on utilizing Kirchoff’s hypothesis and von Karman’s strain–displacement relations. Applying the modified Galerkin’s method to the governing nonlinear partial differential equations, a modal equation of Duffing’s type is obtained and solved by exact integration.

Ray and Shivakumar [102] analyzed active composite layer damping (ACLD) of geometrically nonlinear transient vibrations of laminated thin composite plates using piezoelectric fiber reinforced composite materials. The Golla-Hughes-McTavish (GHM) method had been used to model the constrained viscoelastic layer of the ACLD treatment in the time domain. A finite element model was developed for the cross ply and antisymmetric angle ply plates undergoing geometrically nonlinear vibrations. For deriving coupled electromechanical nonlinear finite element model, von Karman type nonlinear strain displacement relations and the FSDT were used. Singh et al. [2009] studied the post buckling load response of laminated composite plates supported on linear elastic foundation with random system properties using HSDT with von-Karman nonlinear strain-displacement relations. A finite element method is used for spatial discretization of the laminate. Tian et al. [104] proposed a yield criterion that is related to the spherical stress tensor is proposed to describe the mixed hardening of damaged orthotropic materials based on the elasto-plastic mechanics and continuum damage theory. The finite difference method and the Newmark–? method are adopted to make the undetermined variables discretized in the space and time domains respectively. Trindade and Benjeddou [105] presented a analysis of methodologies to evaluate the effective electromechanical coupling coefficient for structures with piezoelectric elements. Finite element method is used to modeling of the electric boundary conditions and comparisons between numerical, analytical and experimental results for beams with bonded extension and embedded shear piezoelectric materials are carried out. Umesh and Ganguli [106] developed a finite element model for a smart composite plate with matrix cracks and studied the effect of matrix cracks in a cantilevered smart composite plate with different electrical and mechanical loadings for several different laminate types and ply angles. Balamurugan and Narayanan [107] extended previous work of Balamurugan and Narayanan [68] to developed a general finite element formulation of stiffened shell structures with distributed piezoelectric sensors and actuators.

Chandrashekar and Ganguli [108] investigated the nonlinear vibration analysis using a C0 assumed strain interpolated finite element plate model based on Reddy’s third order theory. The variance of linear and non linear natural frequencies of the plate due to randomness in its material properties is obtained using Monte Carlo Simulation with Latin Hypercube Sampling technique. Yiming et al. [109] presented a nonlinear model for active vibration control analysis of cross ply piezoelectric laminated plates containing the damage effect of the intra-layer materials and inter-laminar interfaces by using the Von Karman type of nonlinear strains. The active control of damping is derived to the nonlinear dynamic equations and used to actively control the vibration response of the plate by using the Hamilton variation principle and the simple negative velocity feedback control algorithm. Farhadi and Hashemi [110] investigated the feasibility of using piezoelectric patches for multi-mode vibration control of moderately thick rectangular plates based on a finite element formulation. An active damping controller is designed using modal velocity feedbacks for suppression of plate vibrations. Kerur and Ghosh [111] presented a coupled electro-mechanical finite element formulation for active control of geometrically nonlinear transient response of laminated composite plate is studied using FSDT and von Karman type non-linear strain displacements. The Newton Raphson iterative method in association with Newmark time integration method is used to solve the nonlinear finite element equilibrium equation and negative velocity feedback control algorithm is used to control the dynamic response of the smart laminated composite plate.

Sarangi and Ray [112] investigated the performance of the active constrained layer damping (ACLD) treatment in which the constraining layer is made of the vertically reinforced 1-3 piezoelectric composite in time domain for active damping of nonlinear transient vibrations of laminated composite plates using three dimensional finite element model. Dash and Singh [113] developed a probabilistic procedure especially for highly nonlinear problems to obtain mean and standard deviation of the nonlinear natural frequency of the smart laminated composite plate having random material property. Her and Lin [114] investigated the vibration response of a simply supported composite laminated plate excited by piezoelectric actuators and obtained an analytical solution of the vibration response of a simply supported laminated rectangular plate under time harmonic electrical loading. Phung-Van et al. [117] extended formulation based on a cell-based smoothed discrete shear gap element (CS-FEM-DSG3) presented by Nguyen-Thoi et al. [116] and Bletzinger et al. [115] to investigate the static and free
vibration analyses and dynamic control of composite plates integrated with piezoelectric sensors and actuators. The electric potential is assumed to be a linear function through the thickness for each piezoelectric sub-layer. In case of active vibration control of the static deflection, a displacement and velocity feedback control algorithm is used and closed loop control is used to evaluate dynamic response of the plates. Shivakumar et al. [118] studied the geometrically nonlinear vibration control of smart laminated composite panels using the patches of ACLD treatment, which is made of horizontally reinforced PFRC material using three dimensional finite element model.

3.2. Vibration Control of Piezolaminated FGM Plates

He et al. [119] presented a finite element formulation based on the CLPT for the shape and vibration control of the FGM plates with integrated piezoelectric sensors and actuators. The properties of the FGM plates are functionally graded in the thickness direction according to a volume fraction power law distribution. A constant velocity feedback control algorithm is used for the active control of the dynamic response of the FGM plate through closed loop control. The static and dynamic responses are presented for an FGM plate of aluminum oxide/Ti-6Al-4V material composition. Liew et al. [120] presented a finite element formulation based on FSDT for static and dynamic piezothermoelastic analysis and active control of FGM plates subjected to a temperature gradient using integrated piezoelectric sensor/actuator layers. A constant displacement-cum-velocity feedback control algorithm is applied to provide an active feedback control of the integrated FGM plate in a self monitoring/controling system for the bending control and the torsional control. Oo-tao and Tanigawa [121] developed the theoretical formulation of a control of the transient thermoelastic displacement for a FG rectangular plate bonded to a piezoelectric plate due to nonuniform heat supply. The numerical study is carried out for a FG rectangular plate mixture of zirconium oxide and titanium alloy, bonded to a piezoelectric plate of a cadmium selenide solid.

Liew et al. [122] Finite element model based on CLPT is presented for static and dynamic piezothermoelastic analysis and active control of FGM plates under temperature gradient environments using integrated piezoelectric sensor/actuator layers. The properties of FGM plate are in the thickness direction according to a volume fraction power law distribution. FGM plate consists of zirconia and alumnum. Huang and Shen [123] presented the nonlinear free and forced vibration analyses for simply supported, hybrid FGM plate subjected to the combined action of transverse dynamic, electric and thermal loading. The substrate FGM layer consists of zirconia and aluminum. The formulations are based on HSDT and general von Karman type equations and include thermo-piezoelectric effects. Analytical solutions have been presented by using an improved perturbation technique. Kargarnovin et al. [124] analyzed a FGM rectangular plate which is bonded with piezoelectric rectangular patches on the top and/or bottom surface as an actuators/sensors. The governing differential equations of the motion are derived using CLPT and constant electric charge. The solution for the motion equation is obtained using a Fourier series method. Reddy and Ray [125] investigated the optimal control of FG plates using a distributed actuator made of the PFRC material proposed by Mallik and Ray [126] and a distributed monolithic piezoelectric sensor layer. The optimal controller developed by employing the linear quadratic regulator design with output feedback. Ebrahimi and Rastgo [127] investigate the free vibration behavior of circular FG plate integrated with two uniformly distributed actuator layers (PZT4) on the top and bottom surfaces of the circular FG plate based on the CLPT. The material properties of the FG substrate plate are assumed to be graded in the thickness direction according to the power law distribution in terms of the volume fractions of the constituents. The differential equations of the motion are solved analytically for clamped edge boundary condition of the plate. Ebrahimi et al. [128] presented a free vibration analysis of moderately thick circular FG plate integrated with two thin piezoelectric (PZT4) layers based on Mindlin plate theory. The investigation is carried to show the effect of varying the gradient index of FG plate on the free vibration characteristics of the structure.

Ohadi and Fakhar [129] investigated the large amplitude vibration control of FGM plates under thermal gradient and mechanical loads using piezoelectric sensor/actuator layers and presented the nonlinear finite element formulations based on the third order shear deformation theory. A proportional derivative (PD) feedback control algorithm is employed in the analysis and studied the effects of controller gains and presence of noise in sensor voltage on the controller performance. Ebrahimi and Rastgoo [130] presented nonlinear vibration analysis of thin circular pre-stressed FG plate integrated with two uniformly distributed piezoelectric actuator layers with an initial nonlinear large deformation. The nonlinear governing equations of motion are derived based on CPT with von-Karman type geometrical large nonlinear deformations. The initial stress state and pre-vibration deformations of the FG plate that is subjected to in-plane forces and applied actuator voltage is first solved then derived the differential equations that govern the nonlinear vibration behavior of pre-stressed piezoelectric coupled FGM plates by adding an incremental dynamic state. Ebrahimi et al. [131] presented a theoretical model for geometrically nonlinear vibration analysis of piezoelectrically actuated circular FGM using Kirchhoff’s Love hypothesis with von-Karman type geometrical large nonlinear deformations. The FGM plate is the mixture of alumina and aluminum. Panda and Ray [132] presented a geometrically nonlinear dynamic
analysis for FG plates integrated with a patch of active constrained layer damping (ACLD) treatment, which is subject- ed to a temperature field. The temperature field is assumed to be spatially uniform over the substrate plate surfaces and varied through the thickness of the host FG plates and assumed temperature dependent material properties of the FG substrate plates in the analysis.

Panda and Ray [133] presented the geometrically non- linear dynamic analysis of FG laminated composite plates integrated with a patch of (ACLD) treatment. The constraining layer of the ACLD treatment is considered to be made of the piezoelectric fiber reinforced composite (PFRC) material. The constrained viscoelastic layer of the ACLD treatment is modeled using the Golla-Hughes-McTa-vish (GHM) method. Finite element model has been developed to model the open-loop and closed-loop nonlinear dynamics of the overall FG laminated composite plates which are based on the FSDT. Xia and Shen [134] presented the nonlinear free and forced vibration analyses for simply supported, FGM plates with fully covered PFRC actuators subjected to the combined action of transverse dynamic, thermal and electric loads. The material properties of both FGM and PFRC layers are assumed to be temperature dependent. The formulations are based on HSDDT and general von Karman-type equation. Hashemi et al. [135] presented an analytical method to analyze the vibration of piezoelectric coupled thick annular FG plates subjected to different combinations of soft simply supported, hard simply supported and clamped boundary conditions at the inner and outer edges of the annular plate on the basis of the Reddy’s third order shear deformation theory. The properties of host plate are graded in the thickness direction according to a volume fraction power law distribution. The differential equations of motion are solved analytically for various boundary conditions of the plate.

Fakhari and Ohadi [136] developed finite element formulation to investigate geometrically nonlinear vibration behavior of FGM plate with surface bonded piezoelectric layers under thermal, electrical and mechanical loads using HSDDT. The von Karman nonlinear strain-displacement relationship is used to account for the large deflection of the plate. The two control algorithms are employed to control large amplitude vibration control of FGM plate as, classical displacement velocity feedback control and robust H2 control. Yiqi and Yiming [137] analyzed the nonlinear dynamic response and active vibration control of the piezoelectric FG plate based on HSDDT and elastic piezoelectric theory, the nonlinear geometric and constitutive relations of the piezoelectric FG plate are established, and then the nonlinear motion equations of the piezoelectric FG plate are obtained through Hamilton’s variational principle. The nonlinear active vibration control of the structure is carried out with adoption of the negative velocity feedback control algorithm. Fakhari et al. [138] developed finite element formulation based on HSDDT to analyze nonlinear natural frequencies, time and frequency responses of FG plate with surface-bonded piezo- electric layers under thermo-electro-mechanical loads. The von Karman nonlinear strain-displacement relationship is used to account for the large deflection of the plate. The material properties of FGM are assumed temperature dependent. Shirazi et al. [139] studied active vibration control of a simply supported rectangular plate made from FGMs with fuzzy logic control. Modal analysis was implemented to obtain the first nine natural frequencies and mode shapes of the plate.

Ebrahimi [140] presented a nonlinear dynamics and vibration analysis on pre-stressed FG circular plates that are bonded with piezoelectric actuator layers in a thermal environment. The equations of motion are derived using CLPT based on Kirchoff’s-Love hypothesis with von-Karman type geometrical large nonlinear deformations. The material properties of the FG core plate are assumed to be graded in the thickness direction according to the power law distribution. Jadhav and Bajoria [141] investigated the active vibration control analysis of thick FG plate integrated with piezoelectric layer at top and bottom face using finite element method based on FSDDT and HSDDT, von- Karman’s hypothesis and degenerated shell element. The analysis is carried out on newly introduced metal based FGM material which is mixture of aluminum and stainless steel. Jadhav and Bajoria [142] developed a finite element model for free and forced vibration analysis of a piezoelectric thin FGM plate using HSDDT. The natural frequencies are compared with the natural frequencies of a layered composite plate with 21 layers using ANSYS. Jodaei et al. [143] studied the free vibration analysis of FG piezoelectric annular plates using state-space based differential quadrature method and comparative behavior modeling by an artificial neural network. Talabi and Saidi [144] developed a novel exact closed-form procedure based on the third order shear deformation plate theory to analyze in-plane and out-of-plane frequency responses of circular/annular FGM plates embedded in piezoelectric layers for both close/open circuit electrical boundary conditions.

4. CONCLUSIONS

This article makes an effort in summarizing recent development in stability as well as vibration control analysis of piezolaminated composite and FGM plates. Piezoelectric materials possesses a property of direct and converse piezo- electric effects which can be adequately employed to control the deflection, vibration, shape and buckling of the structure. The review paper is divided into following sections: stability analysis of piezolaminated composite plates and piezolaminated FGM plates, vibration control of piezolaminated composite plates and piezolaminated FGM plates. This paper is an evidence of contributions from researchers for high quality and tremendous work in the area of stability as well
as vibration control analysis of piezolaminated composite plates and piezolaminated FG plates.

5. REFERENCES


A Review on Stability and Vibration Control of Piezolaminated Composite/FGM Plate Structures


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