Scheme of Direct Time-domain Extrapolation in Half-space by FDTD Method

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Abstract
A scheme of direct time-domain extrapolation to far fields in half-space by finite-difference time-domain (FDTD) method was proposed, called the time-domain three-wave extrapolation that was in agreement with the nature of FDTD. The result for scattering from a perfectly conducting circular cylinder in half-space validated the proposed scheme.

Keywords
FDTD Method; Half-space; Time-domain Extrapolation

Introduction
The finite-difference time-domain (FDTD) method holds the advantage of calculating all of the frequency components of the scattered field illuminated by a broadband transient pulse simultaneously. In free space, this technique calculates the time-domain response of the scattered field directly, and then broadband frequency response is obtained by carrying out a Fourier transform, which has been developed in the case of 3D (Yee et al., 1991; Taflove et al., 2005) and 2D (Luebbers et al., 1992; Garcia et al., 2000) respectively.

In half-space, however, FDTD is usually used to calculate the far-zone electromagnetics scattered field in various directions at a single frequency by the equivalent electromagnetic currents in near-zone (Wong et al., 1996; Demarest et al., 1996), which can be called the steady state calculation. Being a time-domain scheme, it is natural to calculate the time-domain response even for half-space case, but this result is seldom today. To our knowledge, the direct time-domain extrapolations to far-field for half-space case have been given by Martin et al. (2001) and Capoglu et al. (2006). The former is inherently a frequency method and the other follows the algorithm by using the complicated convolution.

This paper, on the basis of Snell’s law (Harrington, 2001), proposed a new direct time-domain extrapolation scheme for FDTD in half-space for two-dimension case, and this scheme is consistent with the on-the-fly calculation of far field such as that proposed by Luebbers et al. (1992) and Garcia et al. (2000).

Scheme
Shown in Fig. 1 is the on-the-fly calculation scheme for far field in the upper space. Let the half-space model constructed of the upper media (Media 0, μ₀, ε₀) be free space and the lower media (Media 1, μ₁, ε₁) be a lossless dielectric, cᵢ be the velocity in Media i and ϕ be the azimuth to far-zone point of P. For simplicity, the origin of y-axis was supposed to be on the interface. It was assumed that the transformation surface S straddles the interface and Aᵢ be at the computational grid of (l·Δx, m·Δy) on S in Media i, and the wave vector of incident wave be kᵢ. Fig. 1(b) and (c) also show that Oᵢ be the point of reflection and transmission respectively, which is used to be the projected point on the observation time in ϕ-direction, and lᵢ₀, lᵢ₀₀ and tᵢ₀₀₀ are the projection of Aᵢ₀, Oᵢ₀ and Oᵢ₀₀₀ on this time.

It can be seen that the far-field is composed of three components which are the fields radiated by equivalent source in upper space following the direct wave path and the reflected wave path and that in lower media following the transmitted wave path. The on-the-fly calculation following the direct wave path is the same as that in free space (Luebbers et al., 1992; Garcia et al., 2000), and the difference of this calculation between half-space and free-space lies in that following the later two paths, which should be modified. In what follows, the modification of on-the-fly calculation following the reflected and transmitted
wave paths would be illustrated.

\( A_0 \)

(a) FOLLOWING THE DIRECT WAVE PATH

\( \theta \)

\( k \)

\( t_{40} \)

\( x \)

\( y \)

\( \phi \)

\( P \)

\( As \)

Media 0

\( \varepsilon\mu_p \)

\( \varepsilon\mu_h \)

\( S \)

Media 1

\( x \)

\( \theta \)

\( k \)

\( t_{40} \)

\( \Delta t_{40} \)

\( \phi \)

\( p \)

\( P \)

\( A_0 \)

\( t_{40} \)

\( \Delta t_{40} \)

\( \phi \)

\( P \)

\( A_0 \)

\( \phi \)

\( P \)

\( A_0 \)

\( \phi \)

\( P \)

(b) FOLLOWING THE REFLECTED WAVE PATH

\( \theta \)

\( k \)

\( t \)

\( y \)

\( x \)

\( \phi \)

\( \theta \)

\( t \)

\( p \)

\( P \)

\( A_0 \)

\( \phi \)

\( \theta \)

\( t \)

\( 0 \)

\( \phi \)

\( P \)

\( A_0 \)

\( \phi \)

\( P \)

\( A_0 \)

\( \phi \)

\( P \)

(c) FOLLOWING THE TRANSMITTED WAVE PATH

FIG.1 THE ON-THE-FLY CALCULATION OF FAR FIELD IN HALF-SPACE

a) The modification of projection

It is nearly impossible to project Point \( A \) on the observation time directly by the reflected and transmitted wave paths, but which can be realized indirectly by Point \( O_{ai} \) as shown in Fig. 1(b) and (c). Considering \( t_{ai} \) is not the true projection of Point \( A \), and the true that should be delayed with associated time of \( \Delta t_{ai} = \frac{AO_{ai}}{c} \), which can result in that the true projection is \( t_{ai}' = t_{ai} + \Delta t_{ai} \). Where \( AO_{ai} = \frac{m \cdot \Delta r \cdot \cos \phi}{\mu_\phi} \) and \( AO_{ai} = -\frac{m \cdot \Delta r \cdot \cos \theta}{\mu_\phi} \) are the distance of Point \( A \) to Point \( O_{ai} \), the coordinates of Point \( O_{ai} \) and Point \( O_{ai} \) are \( (l \Delta x + m \Delta y \cdot c \tan \phi, 0) \) and \( (l \Delta x - m \Delta y \cdot \tan \theta, 0) \), and \( \theta = \arcsin \left( \frac{\mu_\phi}{\mu_\phi} \cdot \cos \phi \right) \) is the transmission angle derived by Snell’s law.

b) The modification of scale factor

The radiation field to the equivalent source at Point \( A \) following the direct wave path contributes to the far-field completely. However, the contribution following the reflected and transmitted wave path should be modified by the scale factor of Fresnel reflection coefficient \( (R) \) and Fresnel transmission coefficient \( (T) \) (Harrington, 2001) respectively. \( R \) and \( T \) have different expression for different polarized wave. It’s very worth noting that the transmission should be from Media 1 to Media 0.

Validation

There are many half-space results of RCS versus azimuth by running steady state calculation. In this letter, such an example was listed as shown in the reference (Borghi et al., 1996) to validate the scheme, in which a perfectly conducting circular cylinder with the radius of \( a \) is at a distance \( h \) from the interface. The relative permittivity of the dielectric in lower space is 4, i.e., the refractive index \((n)\) is 2, which is illuminated by a TM polarized Gaussian pulse with \( \theta_a = 90^\circ \) and \( \theta_a = 120^\circ \). The FDTD code uses the following date:

1. \( a = 0.5m \) and \( h = 1m \);
2. \( \delta = \Delta x = \Delta y = 0.02m \);
3. \( \Delta t = \delta/(2c_0) \);
4. Gaussian pulse \( \exp\left(-\frac{4\pi(t-t_0)^2}{\tau^2}\right) \) with \( \tau = 60\Delta t \) and \( t_0 = 0.8\tau \).

The validation is developed by following method:

1. Perform an FDTD calculation in many directions and Fourier transform to obtain the broadband frequency domain response of far-field in those directions.
2. Extract the values at one same frequency from the frequency domain response in various directions and obtain the steady state result as shown in reference (Borghi et al., 1996).

Fig. 2 shows the curves calculated by the scheme in this letter with \( \theta_a = 90^\circ \) and \( \theta_a = 120^\circ \), where (a) are the time domain response in the directions of \( \theta = 30^\circ \),
90° and 150°, (b) are the corresponding broadband response by using the Fourier transform, and (c) are the extracted data in various directions for 300MHz (meeting \( ka = \pi \) and \( kh = 2\pi \)). In the case of \( \theta_{in} = 90° \), the response in \( \varphi =30° \) and \( \varphi =150° \) are entirely coincidence for the symmetry of the model and the incident wave direction.

Fig. 2(c) can be compared with the dashed curves in Fig. 6 with \( n=2 \) reported by Borghi et al. (1996). The results are identical except that there is no specified information of amplitude in the latter case.

**Conclusion**

A direct time-domain extrapolation to the far field in half-space to calculate the broadband response has been proposed based on the theory of electromagnetic wave propagation, which was the improvement of the method proposed by Luebbers et al. (1992) and Garcia et al. (2000), named as the time-domain three-wave extrapolation. Results have been obtained to validate the scheme in this letters.

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